THE SHAPE OF REALITY:

RESTORING OUR FAITH IN THE UNITY, COHERENCE, AND COMPLETENESS OF CREATION

The Shape of Reality

Loyd L. Fueston, Jr.

Published by Loyd L. Fueston, Jr.

My soul [in Thomas the organ for thought] is not I; and if only souls are saved, I am not saved, nor is any man.

[From the *Commentary to 1 Corinthians* by St. Thomas Aquinas as quoted by Hannah Arendt in *The Life of the Mind* (page 43).]

Modernity is not simply a historically-datable cultural phenomenon; in reality it requires a new focus, a more exact understanding of the nature of man.

[Pope Benedict XVI, in a speech given on 2008/06/07 to participants in the sixth European Symposium of University Professors, which was held in Rome from 2008/06/04 to 2008/06/07 on the theme: "Broadening the Horizons of Reason. Prospects for Philosophy".]

Contents

List of Figures

Preface

Is a man a unity or is he a collection of warring fragments? Or is the proper question perhaps my favorite: Is man some sort of complex being not describable in current terms of discourse? I should qualify my statement of the last possibility, which I believe to the closest to the truth: I think mathematicians working in the most abstract regions currently accessible to the human mind have discovered tools of thought, quantitative and qualitative, which can provide us with superior ways of discussing complex forms of created being. That entire argument is one which can be carried out only by way of a program to show such is the case. This book will be the first step in such a program—unless this step sends me into a brier-patch or over a cliff.

But even a giant step wouldn't be enough. We need to explore ahead of where we can reach; that exploration will necessarily take place by way of the human imagination, rationality unleashed rather than rationality constrained by tight rules. This is a project I've been working on for 25 years, if not always consciously so. I've explored in the way of a novelist and even (quite) amateur poet through novels including some in which characters were poets—if questionably so; I've explored in the way of an admirer of mathematics and science through essays; I've explored in the way of a philosopher and theologian through other essays often overlapping with those dealing with mathematics and science.

Years ago, I conceived the possibility of adopting the insights of the most fundamental of the sciences, mathematics, to the development of a worldview adequate to the needs of man in the 21st century. This book will be a step in that direction. Note that the epigraphs for this book are the same as my earlier book, *A More Exact Understanding of Human Being* [14]. I'll quote them again for the ease of the reader:

> My soul [in Thomas the organ for thought] is not I; and if only

> souls are saved, I am not saved, nor is any man. [From the *Commentary to 1 Corinthians* by St. Thomas Aquinas as quoted by Hannah Arendt in *The Life of the Mind* (page 43).]
>
> Modernity is not simply a historically-datable cultural phenomenon; in reality it requires a new focus, a more exact understanding of the nature of man. [Pope Benedict XVI, in a speech given on 2008/06/07 to participants in the sixth European Symposium of University Professors, which was held in Rome from 2008/06/04 to 2008/06/07 on the theme: "Broadening the Horizons of Reason. Prospects for Philosophy".]

To make one aspect of the above claim more clear, I'll provide another quote included in the body of *A More Exact Understanding of Human Being* [14]:

> [J]ust as a disciple reaches an understanding of the teacher's wisdom by the words he hears from him, so man can reach an understanding of God's wisdom by examining the creatures [God] made... [Page 17 of St. Thomas Aquinas' commentary on 1 Corinthians.] [2]

It seems to me, and has seemed to me for years, that the greatest weakness we bring to the task of understanding created being is our inability to see a human community (the Body of Christ in the limit) as being one without taking away from the individuality of the members of that community. We moderns think ourselves to be some sort of gathering of isolated individuals or else we are a collective with compromised or even deleted individuality.

I wish to provide rational ways of describing and understanding of human communities as being real.

Acknowledgments

I would like to thank Wikipedia, its administrators, and its writers and editors of articles in mathematics. I have drawn upon articles—mostly on an undergraduate college level—for definitions of standard concepts or entities in the fields of differential geometry and topology as well as the foundational field of calculus. It was a great help to have good quality and standardized material to draw upon. They, of course, bear no responsibility—not even the least bit of guilt for my use of this material.

Introduction

Human being isn't *merely* quantifiable but many components or aspects of human being are quantifiable. Other components or aspects are more describable by the term 'qualitative', but this shouldn't be taken as connoting 'irrational' or 'mystical'. Our understanding of the qualitative aspects of being, human and other, should be as orderly as our understanding of the quantitative aspects of being. Yet, I'll warn that I threaten by the end of this book to pull off one of those sleights of hand for which philosophers are famous—I'll suggest the terms 'qualitative' and 'quantitative' should be retired in favor of 'abstract' and 'concrete'. Since these latter terms are understood in ways I think to be implausible in light of modern mathematics and physics, I'll use the less plausible terms, 'qualitative' and 'quantitative' for much of the book while gently banging on the drum of 'abstract being'.

So, how are we to see our human beings as unified, even if only in principle? Have the quantitative and the qualitative come to be "the new dualism"? Some Medieval Scholastics rejected the concept of 'soul' or 'mind' because it's not clear if it be at all possible to unite two different sorts of substances to make a single human being. Aquinas took a different route and posited a mind as described by Hannah Arendt in the quotation above:

> My soul [in Thomas the organ for thought] is not I; and if only souls are saved, I am not saved, nor is any man. [From the Commentary to 1 Corinthians 15 by St. Thomas Aquinas as quoted by Hannah Arendt in *The Life of the Mind* [3] (page 43).]

In other words, the Thomistic soul (or mind), said by Hannah Arendt to be something you would not wish to be, was a non-human entity attached to a human being as a cochlear implant is attached. You could almost joke Aquinas was the first prophet of cyborgs. It was clear that all that was

truly human to Aquinas was enfleshed, embodied. This made sense given Aquinas' difficulty in seeing how the 'inert' matter of the brain could be dynamic enough to engage in conceptual thought. I would have to think he longed for a way to see higher-level rational thought, especially thought about abstractions, to be truly an activity of the embodied human being. We can now meet that longing:

- The matter of the brain, indeed all matter, is a very dynamic sort of stuff, a partially frozen form of energy which constantly interacts with hotter forms of stuff, hot energy and fields.
- The brain is the result of evolutionary and developmental processes of a complex and sometimes nasty sort.
- The entire universe is the result of a variety of processes which can be roughly and accurately classified as evolutionary and developmental.
- Matter forms, in some sense, by what can be described as a collapse of a quantum wavefunction, that is, concrete forms of being are shaped from strange, abstract forms of being.

This last item is disturbing, even if the term 'collapse' is rejected as some philosophers and scientists recommend—without having any less controversial ways of speaking about that strange event. How can matter be made from some form of being describable as a 'wavefunction', a mathematical equation?

Thinking in usefully simplistic terms by way of stealing ideas from very sophisticated realms of thought in physics:

> Existing theories of gravity combine space and time into a single structure of spacetime in which time is added as a fourth dimension to the three-dimensional Euclidean structure of space. Then we find that space and time can be deformed by interactions with matter which is itself describable in terms of the quantum mechanical wavefunction. Yet, many physicists and at least some others who think deeply on these matters are convinced that all this can be combined into some sort of unified theory of physical stuff or of truly everything for those willing to define everything as what can be described in terms

> of physics. All sorts of entities of different stuff and even non-stuff are combined in an as-yet unconstructed grand—or perhaps grandiose—theory of 'everything' or TOE. This may well prove to be a scientific version of the mistake often made by generals and politicians: preparing to fight the previous war. Those physicists see TOE as a culmination of the extraordinarily successful efforts to construct a standard model of physical particles and most of the forces between those particles, but gravity has remained resistant to such efforts, yet it is surely an enemy which will fall to the same weapons and strategies and tactics as used successfully in the last war.

In any case, I've proposed a principle in past writings: Being is being is...

I've shortened the principle for poetic reasons; being in that claim is actually "created being". And I'll be mostly avoiding the whole issue of God's being is immensely complex for intelligent discussions by atheologians as well as Christian and Jewish theologians, though there are clearly theological implications of my way of thinking about being and how it forms complex entities. Those implications are particularly clear, and exceedingly dangerous, when dealing with our understanding, of lack of understanding, of the Holy Trinity of Christian belief. I'll mostly write about created being, a constraint which will clearly hold even when I use 'being' rather than 'created being'.

I think created being, stuff and non-stuff, alike can be tied together into a coherent understanding by way of modern mathematics which has developed some powerful tools which are useful in dealing with various concrete situations, engineering and scientific, and can also be used for understanding quantitative and qualitative aspects of what I call abstract being—the non-stuff, or at least non-thing stuff, of modern algebra and topology and category theory and so on, as well as the non-thing stuff of moral character or soul or some aspects of mind and just the general feeling of order and goodness which often emanates from what lies around us. Can we somehow use these tools to provide an understanding of human being which gives proper room and respect to the various aspects of human being, qualitative and quantitative, individual and communal? I think so and I'm writing this book as an experiment of sorts to see if I can produce at least a workable start on such an understanding.

Will I be providing some hi-tech answer to the question, "What is a human being?" No, though I will provide some ways to see various answers, hi-tech and other, as potentially different 'mappings' of the same entity. Mappings, or functions, is a mathematical term I'll use, perhaps abuse in the eyes of some, in providing ways of describing and understanding qualitative and quantitative aspects of human being.

Will I be providing even a tentative model of human being which allows prediction of the future or at least accurate and precise modeling of the past? No, but I'll be pointing out that even qualitative aspects of human beings result in physical evidence which can be quantified if only by way of chronologies.

I will be drawing upon some basic ideas from mostly differential geometry and topology in this book to provide a set of concepts and words which allow us to discuss human being as a coherent entity with quantitative and qualitative parts and aspects, as well as individual and communal parts and aspects. I have tentative plans for future books which will use algebra and other mathematical fields to improve my discussions by further enriching and complexifying my understanding of created being. Those books might well be written by some younger thinkers with greater knowledge of modern mathematics than I have.

Part I

Laying Some Foundations

1 Reuniting Quantitative and Qualitative Human Thought

In order to better understand this complex world of concrete and abstract being, things with both quantitative and qualitative aspects, entities which are individual (such as a man or woman or child) but also communal (substantially and not nominally members of religious and political and cultural and economic communities), I'll be drawing on some simple levels of modern mathematics though mostly topology (qualitative geometry) and differential geometry. Some of the work done in these fields of study, certainly at the introductory or undergraduate level, is quantitative and formal in a way that might look frighteningly familiar to those who took geometry or algebra in high school.

Mathematical thought exploded, but in a surprisingly sustainable way, since the time of Newton or so. A little more recently, maybe since 1800 or so, various fields developed lines of thought which are far more qualitative. And other fields rose up, including Symplectic geometry and category theory, which could be qualitative in sometimes strange and extreme ways.

Sometimes, mathematical fields of study can be semi-quantitative, a true strangeness. As an example of this seeming paradox, 'semi-quantitative', symplectic geometry deals with spaces where geometric entities have area, preserved under certain important transformations, but don't have quantifiable lengths or angles. These are not spaces of utterly abstract natures and most certainly do they have practical applications in physics and engineering. One such space is that of phase spaces or the closely related state spaces. For example, position and momentum in both classical and quantum mechanics form such a space in which areas are always measurable but not lengths or angles. For those who took at least high school algebra, think of position as the x-axis and momentum as the y-axis but remember

the strangeness of areas being always measurable but not lengths or angles.

Sometimes, mathematical fields of study can even be outright qualitative, at least in part. In *Notes on Introductory Point-Set Topology* [17], Allen Hatcher tells us: "One way to describe the subject of topology is to say that it is qualitative geometry." This doesn't necessarily point to anything exotic. In *Topology and Geometry for Physicists* [22], we learn: "[T]opology produces theorems that are usually qualitative in nature—they may assert, for example, the existence or non-existence of an object. They will not in general, provide the means for its construction." Sounds almost like theology and metaphysics.

My goal is to be able to think and speak and write of human animals and their communities, of human being in its individual and communal forms, so that these human beings of various types potentially, though imperfectly so, meet the criteria I've suggested in my past writings for the definition of certain types of complex entities which have narrative context as well as physical being and relationships to various other forms of physical beings. These criteria are unity and coherence and completeness, all within the context of what human being is and can be extrapolated to be. Worlds, such as this universe seen in the context of a moral order, and also human communities are entities which in various ways and to various degrees meet such criteria. In simpler—and potentially misleading—terms, I'm distinguishing between different levels of concrete, thing-like being in a manner similar to that of physicists in their levels of elementary particles (electrons, photons, quarks, etc), composite subatomic particles (protons, neutrons, mesons), atoms, molecules, and then various levels of entities which are composites of particles. As a brutal summary, those various levels of entities range from such things as interstellar clouds of ionized hydrogen to rocks hurtling through space to planets to stars to galaxies to various groupings of galaxies. Somewhere in that hierarchical listing, we might find retroviruses, viruses, bacteria, the domain of eukaryotes which includes fungi and plants and animals.

I'm throwing a lot of general ideas at you the reader to prepare you for arguments that a lot of this can be understood in more general and more profound terms than scientistic thinkers would claim. This doesn't mean terms that exclude quantitative knowledge or the related sciences. I intend to include physics and chemistry and biology, as well as all the 'softer' sciences such as economics or politics or sociology. What is needed is a more complete understanding of created being in its quantitative and qualitative

parts and aspects, not an alleged qualitative understanding to run parallel to the better developed quantitative understandings developed slowly from the time of the early Greek philosophers, a little faster as optics and kinematics were pretty well-developed along with such concepts as impulse (defective but interesting try at momentum) and (almost) mathematical limits in the Middle Ages. (See *Medieval Cosmology* [6] for an interesting look at the accomplishments of Medieval scientists, including some very important near-misses such as the mathematically almost-rigorous concept of limit which appeared in some Medieval thought.)

I'll be throwing much more at you the reader, but don't get frightened if you don't feel your self to be mathematically inclined. There isn't going to be anything such as the construction of a mathematical model of a human being. There will be a discussion of a very general model of a human being which will have both quantitative and qualitative aspects, based upon generalizations of powerful mathematical ways of understanding entities and their relationships. This will perhaps provide a foundation for future work which might produce more explicit models of human being in terms of both quantitative and qualitative being. It will also provide for similar views of created being in general.

Before going on, I'll note a comment from a highly regarded cosmologist—a physicist who studies the universe as an entity in its own right though he certainly must and does consider its contents such as stars and black holes. In *Cosmology: A First Course* [20], Marc Lachièze-Rey writes:

> The best definition of the cosmos is possibly the ensemble of relations between the constituent parts of the cosmos.

This is a statement by one scientist of my claim that relationships are primary over stuff, as St John the Evangelist and his followers claimed in their gospel and other writings. The world exists because God first loved it. Each of us exists because God first loved each of us. God doesn't create and then choose to love; He loves and His active love brings into existence what He chooses to love. On the level of creatures who help God to shape what exists and from the Christian viewpoint, we can bring about good changes, those consistent with God's love by praying for and loving another individual or community, even by praying for and loving our currently disordered, anti-Christian country.

My goal in this book and the related weblog posts is to provide ways of speaking rationally and respectfully of created being, especially human being, in its entirety and not just in 'scientific' terms of what can be quantified or in 'poetic' terms of what can't be quantified. The goal most certainly isn't to produce equations to predict what you will do when facing a certain difficult decision, nor will there be an equation which tries to explain—by way of reduction—political or cultural structures strictly in terms of economics, the possibilities in the environment, and a people's realized ways of doing business. There won't even be the hint that such is reliably possible in a statistical manner, though I will say directly that we can identify patterns in the demographic and political and economic structures of a human community, including patterns which should be treated as signals for the possible approach of political and social and economic disaster.

This very general model I'll construct will be more of a patchy and ill-formed sphere [1]. Each of those patches which make up the sphere will be a region which corresponds to an individual human being, but overlaps with the regions which are other individual human beings. The more complete general model would also include spheres with regions which might themselves be lower-dimension spheres or other 'shapes', each of them corresponding to some community, possibly made of sub-communities, or to collections (sets or whatever) of individuals. Those various regions, individuals or smaller communities, will be mappable to other patchy and ill-formed spheres or maybe similar sorts of Euclidean or Cartesian surfaces [2]—think of charts with coordinates of x and y. To make it possible to take the first step, I'll deal only with regions which map to Euclidean or Cartesian planes of the sort studied in high school mathematics courses.

Take it slowly and let such ideas build up in your mind. Think of what I'm suggesting as being analogous to networks of networks. Your office building has a network of computers and telephones, perhaps hooked up to security systems and climate controls for heating and cooling of the air. This office building is then part of a network, perhaps, in an office park, itself part of a network in, say, San Diego itself part of networks including all of California, the United States, and so on. Yet these networks are not hooked up in strictly hierarchical ways. In fact, part of the motivation for

[1] Don't take this too literally but spheres are often used for argument's sake or for pedagogical purposes in advanced fields of study such as topology and the most most exotic forms of geometry.

[2] A n-sphere is the n-dimensional surface of an (n+1)-dimensional 'solid' ball.

the Internet was to get a system in which an arbitrary node of an arbitrary network could connect to an arbitrary node and network by way of a readily available path. You can tavel an arbitrary path if you will in order to avoid too much hierarchical rigmarole. My suggestions are a bit more general than networks, though networks could be in the eventual model.

Essentially, what's possible in the world of human beings, individual and communal, should be possible in any model which is a plausible understanding those human beings. Yet, psychiatric models, as one example, tend to deal with the mental and emotional properties of a human being with ad-hoc explanations added on to account for communal issues. Statistical analyses of tests of human emotional structures ignore all but the immediate test-subject as if she were a plant in a tightly controlled and isolated agricultural experiment [3].

Let's get on and see if something better can be proposed, something more promising as a model for understanding human being (and other complex entities) in all their richness and complexity.

[3] In the real world, any sensitive psychiatrist or other medical professional, any sensitive teacher or clergyman or policeman or—most certainly—parent, would realize the importance of communal relationships, including even the most ephemeral of peer-group relationships

2 Some Preliminary Mathematical Matters

2.1 Introduction

I'll be introducing a few background ideas underlying some mathematical thought of a qualitative sort—think of qualitative in terms of shape for now. We aren't even interested in the area or the length of the perimeter of a circle, only in the fact that it is a circle or very similar to a circle in any of several ways of interest or use to mathematicians or scientists. For example, if a closed doodle lies on a surface, it can—perhaps—be usefully reshaped to a circle on that surface by way of certain types of transformations, such as those which are continuous. Surface should be thought of in general terms of one dimension less than the space in which it is embedded; the surface of a 3-dimensional sphere is a plane with two dimensions while the surface of a 4-dimensional hyper-sphere is a sphere with three dimensions [1].

This should not be considered as an effort to produce some sort of mathematics for poets and theologians but rather an effort to abstract out of mathematics some concepts and tools useful for descriptions of types or aspects of being which are not strictly quantitative. At the same time, we need to retain the ability to describe entities with both quantitative and qualitative parts and aspects. We many not be able to add apples and oranges as such, but we can produce a set of 2 apples and 5 oranges which

[1] Be careful. This is useful language for learning how to think of this, but that higher dimension space may not actually exist. For example, the three 'ordinary' dimensions of this universe are as if they are equivalent to a sphere which is the surface of a four-dimensional ball or perhaps something very similar, but that four-dimensional ball doesn't necessarily exist and, in fact, probably doesn't exist in any meaningful way. In fact, the four-dimensional space which would contain the ball probably doesn't exist.

can be added to produce 7 members of a set, 7 pieces of fruit as it turns out.

The ultimate goal is to produce a disciplined way to describe and discuss and analyze being, raw being and being as shaped into complex entities—especially human being in its individual and communal forms. In older language which is misleading but far from entirely wrong: we need to describe human being which is both body and soul.

Try to develop an imaginative approach to the mathematical ideas which are presented because they are drawn from rigorously derived descriptions of abstract entities and abstract relationships which I'm presenting as useful ways of thinking about being which is not strictly mathematical in the usual (and oversimplified) sense. For example, geometric reasoning has been used to speak of moral matters, the straight path and the crooked man, though too often we don't try to understand what is the reason for a straight path being something like a path of virtue. And we do need to generalize as many might know from discussions in geography courses or even from reading a popular article about the shortest flight path between New York City and London. A straight line on a 2-sphere—the surface of a 3-ball or 3-dimensional ball, such as the earth's surface (approximately), is actually a part of a so-called great circle. If you have a globe handy, pick out two points and place the end of a string at one, stretch it out to the other point so that it's tight. That arc is a straight line on that surface, that 2-sphere, straight in being the shortest distance between the two points on that surface.

> The great-circle or orthodromic distance is the shortest distance between two points on the surface of a sphere, measured along the surface of the sphere (as opposed to a straight line through the sphere's interior). The distance between two points in Euclidean space is the length of a straight line between them, but on the sphere there are no straight lines. In non-Euclidean geometry, straight lines are replaced with geodesics. Geodesics on the sphere are the great circles (circles on the sphere whose centers coincide with the center of the sphere).
>
> Through any two points on a sphere which are not directly opposite each other, there is a unique great circle. The two points separate the great circle into two arcs. The length of the shorter arc is the great-circle distance between the points.

> A great circle endowed with such a distance is the Riemannian circle.
>
> Between two points which are directly opposite each other, called antipodal points, there are infinitely many great circles, but all great circle arcs between antipodal points have the same length, i.e. half the circumference of the circle, or πxr, where r is the radius of the sphere.
>
> The Earth is nearly spherical so great-circle distance formulas give the distance between points on the surface of the Earth (as the crow flies) correct to within 0.5% or so. [See `https://en.wikipedia.org/wiki/Great-circle_distance`.]

Note that the above quote from Wikipedia considers a sphere as a 3-dimensional solid object. I'll stick to the usage of the mathematicians I've relied on for learning topology and differential geometry and use the 2-sphere as the name of the surface of a 3-ball, a solid ball of three dimensions. I'll try to be careful in my own usage of terms and to note when I provide a quote which uses words in a different way.

Terms and concepts, along with other things, can be usefully drawn from mathematics and physics, as those fields of study first borrowed terms and concepts from various common languages, trying to rigorously define and explore them. Besides 'line', 'point', 'function', 'group', and many others in mathematics, we should realize that 'mass', 'force', 'weight', 'dimension', and many other words often have very specific meanings in physical sciences and engineering; the confusion is that much greater because of the significant conceptual overlap. For an example that I'll not explore: 'mass' and 'weight' are nearly synonymous in many common usages but refer to different properties in physics, with those two properties being well-defined. Teachers of high school or freshman physics courses often have to spend surprising amounts of time making it clear that 'mass' isn't the same as 'weight'. I'll move on and leave it to some to remember their own time of confusion about this issue and to some to explore the issue if they wish.

To travel the road I'm scouting, we'll have to think sometimes in terms that are inaccurate or possibly even heading in a slightly wrong direction. So long as you learn to recognize and correct your misconceptions, it's better to learn to think with some freedom and to make mistakes. And it's not so hard to correct your mistakes by reading, thinking, maybe doing some problems or trying to repeat some proofs with the book closed, rereading,

maybe reading a different book on the same subject. Wikipedia (`https://en.wikipedia.org`) is available with lots of information on subjects in mathematics and related fields of study as well as a variety of non-related fields. But there are other good sources of information on the Internet, including *Wolfram Mathworld* at `http://mathworld.wolfram.com/` and the websites of major producers of encyclopedias and some publishers of mathematical and scientific works. A quick check on your favorite search engine might lead you to websites or downloadable books on a subject, written by faculty members at prestigious universities or research centers, sometimes even written by famous mathematicians. The reader should also be aware that even in mathematics and physics, a wider or deeper understanding requires a metaphysics and a theology or atheology. No source is fully trustworthy especially on such fields as politics and history but also controversial and speculative topics in mathematics and science.

The road we wish to travel to start is actually laid-out well by mathematicians and physicists and computer scientists and others. The road we wish to travel ultimately is near that road so well laid-out but it may go through some brush and even leave us at the edge of a cliff or two. We have our work cut out but we can draw upon mathematical work which is very solid.

This first version of this book will be exploratory. There should be additional books, advancing this way of thought a bit and drawing upon abstract algebras, category theory and other very abstract fields of mathematical thought.

2.2 Invariants, Truths Which Create Quantity

Mathematicians and physicists and chemists and other scientists, also those in technological fields, are interested in invariants, in those things or measurements of things which don't change. This is true even of those whose main goal is to study dynamic aspects of reality—you can't really study what is dynamic until you understand what is invariant in the system, perhaps truly invariant in the way of the fundamental physical laws which govern matter and perhaps invariant in some local sense where local might refer to this stage of the evolution of life or this stage in the evolution and

development of stars and galaxies or to the particular sorts of environments in which we Earthlings live.

An invariant can be a single number which becomes the value of a parameter in, for example, the equation which quantifies gravitation force in *Newton's law of universal gravitation*—see `https://en.wikipedia.org/wiki/Newton's_law_of_universal_gravitation`:

$$F = \frac{Gm_1m_2}{r^2} \tag{2.1}$$

where:

F is the force between the masses;
G is the gravitational constant $6.67410^{11} \times m^3 \times kg^1 \times s^2$;
m_1 is the first mass;
m_2 is the second mass;
r is the distance between the centers of the masses.

G seems to be an invariant in our universe, that is, it has an unchanging value of $6.67410^{11} \times m^3 \times kg^1 \times s^2$ where m represents meters, kg represents kilograms, and s represents seconds. On the other hand, some have speculated it changes over time and the invariant would then be a formula, $f(x_0, x_1, x_2, \ldots)$ where x_0 is intended to represent time and the other variables would represent perhaps a variety of other factors known only to God or—perhaps—to some human yet unborn.

Thinking about even the most concrete, thing-like forms of being can be fun, but messy. It's easy to get lost in all that's involved. After all, what's involved in all forms of created being, abstract and concrete.

There is a philosophical, a metaphysical, aspect to invariants, one important in the creation and shaping of a quantitative, concrete world; it helps us to understand the concept of abstract being and how it can be shaped into more concrete forms of being. It is this philosophical aspect which lies behind the title of this section: *Invariants, Truths Which Create Quantity.*

Invariants are sometimes properties of a global realm even if we can see them on a local level. For example, in Einstein's *Special Relativity* (see `https://en.wikipedia.org/wiki/Special_relativity`), we see local measurements, that is—location, (X_1, X_2, X_3) on a conventional 3-dimensional grid) combined with local time (X_0) in the famous formula:

$$ds^2 = -dX_0^2 + dX_1^2 + dX_2^2 + dX_3^2, \tag{2.2}$$

In the above equation, dX_0^2 represents time. The above invariant is a particular formula for measuring distance in a world corresponding to the dynamics of Einstein's Special Theory of Relativity. This sort of measurement is called a metric by mathematician. There are also invariants which are 'lower-level' and are called 'constants', such as parameters in particular equations for physical science. For example, there is the speed of light which constrains the above metric and Avrogado's number from chemistry.

In a strong sense, perhaps more than just an 'approximation', space is 3-dimensional and does flow through time on the local level, but all that local stuff is curved about by the global properties of the universe as a whole and by the global properties of regions of powerful gravitational fields—such as black holes. To fully and accurately describe events affected by great distances or powerful gravitational fields, we would need to utilize Einstein's *General Theory of Relativity*—see `https://en.wikipedia.org/wiki/General_relativity`.

Unfortunately, there doesn't seem to be a general understanding of 'invariant' which abstracts from the various understandings of algebra and other fields of mathematics, physics and other fields of physical science, linguistics and other fields of social science, and so on. An effort to create such an abstraction lies beyond the scope of this work, but I need at least a crude abstraction to a better one. Let me propose such a crude abstraction, which is really part of a better way of describing the processes by which abstract being is shaped to be more concrete, including the perhaps more specific ways by which qualitative being becomes quantitative being:

Invariant A model, formula, parametrization of a formula, or constant which brings the stuff of this universe into a more specific state of being, that is, a state of being more fully quantitative or at least amenable to some quantification.

In other words, I'm labeling as an invariant anything which defines a more particular, more concrete, form of being and which is added to or imposed upon one or more forms of relatively more abstract being. Note that such processes involving such particularization make possible individuality in concrete entities: gravitational processes make possible stars and plan-

ets and the entire universe, the biochemical processes which make possible living cells and multi-cellular organisms.

Let's consider an invariant relationship which many of us studied in high school geometry and trigonometry; some studied this particular relationship further, in a mathematical or scientific or engineering context, college. (There are less particular, less constraining forms of this relationship, but such a discussion would be beyond the scope of this book.)

Here is the famous *Pythagorean Theorem*—see `https://en.wikipedia.org/wiki/Pythagorean_theorem`:

$$c^2 = a^2 + b^2 \tag{2.3}$$

The variables a and b are the distances of the two sides of a right triangle which meet at the right angle and c is the other side, the side which faces the right angle as shown in this figure:

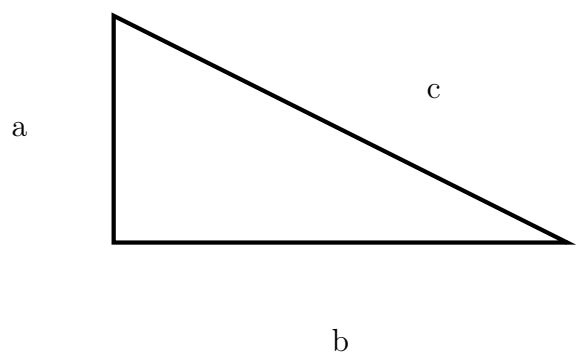

Figure 2.1: Pythagorean Theorem: A Right Triangle

;

The Pythagorean Theorem is true in a flat or Euclidean space, but not generally. It is true when the sum of the angles of a triangle is 180° which is equivalent to saying it is a flat space. On the sphere, the surface of a ball, a three-dimensional solid object [2], the sum of angles will be greater than 180°. On the surface which is shaped like the surface of a saddle, the sum of angles will be less than 180°. Those who are interested can read

[2]Remember that the ball doesn't actually have to exist, as it is very likely there is no four-dimensional ball for the three-sphere which is the three non-time spaces of our universe.

the short article, *Sum of angles of a triangle* at `https://en.wikipedia.org/wiki/Sum_of_angles_of_a_triangle` for a flavor of what is involved in Euclidean vs non-Euclidean geometries and also for a flavor of modern abstract mathematical thought if it is alien to you. Contemplation of this specific issue can, in fact, provide a richer understanding of what I'm getting at in this section and in many sections of this book.

For now, it is sufficient to understand that the Pythagorean Theorem is an invariant relationship which is part of a constraining of a more abstract space to be Euclidean. Some will know or might have guessed that the "Special-relativity" metric in equation 2.2 is some sort of cousin to the "Pythagorean" metric in equation 2.3. Yes, that famous formula from high-school geometry is also a metric.

If we take all this knowledge from mathematics and from mathematical physics and abstract it to 'qualitative' concepts and language, we will have various options for dealing with invariants both quantitative and qualitative. These qualitative invariants will take the form of constraints upon concepts.

As a mathematician might say, we are exploring and describing and not nearly close to the point of producing a tight system of thought, let alone 'proving' any of the important parts.

The stronger and more specific the invariant, the more concrete the environment and the entities it contains. Too strong, too specific the invariants and we have determinism, a fully-determined world and fully-determined human being in the context I'm discussing.

2.3 Space

Space as we think about it in common sense terms is combined with time in modern physics because of **global** properties of this universe. We live in spacetime, not space moving through time. Yet, we perceive that wrongful understanding, space moving through time, in our ordinary lives. We can 'see' spacetime mostly in measurements under extreme conditions, as elementary particles or objects longfar agoway move about in ways consistent with Einstein's reformulation of space moving through time as spacetime.

We should remember that time is intertwined with space but is different from the three dimensions of space. We should also remember that mathematicians and physicists have generalized and abstracted space. The common sense perception of space is part of some branches of geometry as

well as the 'ordinary' work of many scientists and engineers and applied mathematicians:

Space Space is the boundless three-dimensional extent in which objects and events have relative position and direction. [See *Space* at `https://en.wikipedia.org/wiki/Space`.]

That same article goes on to tell us:

> Physical space is often conceived in three linear dimensions, although modern physicists usually consider it, with time, to be part of a boundless four-dimensional continuum known as space-time. The concept of space is considered to be of fundamental importance to an understanding of the physical universe. However, disagreement continues between philosophers over whether it is itself an entity, a relationship between entities, or part of a conceptual framework.

I'll be taking a viewpoint which is pretty much that of the physicists and mathematicians in the sense of just using their understanding of space, including a powerful mathematical abstraction of the concept of space:

Space (mathematics) In mathematics, a space is a set (sometimes called a universe) with some added structure.

Mathematical spaces often form a hierarchy, i.e., one space may inherit all the characteristics of a parent space. [See *Space (mathematics)* at `https://en.wikipedia.org/wiki/Space_(mathematics)`]

When I write about concrete spaces, those we move through, I'll use terms introduced by Einstein and his successors, of locally flat spaces which are part of a globally curved space. This will partly be a pedagogical strategy—the more important idea will be a general one that a complex entity, describable as set in a correspondingly complex space of a mathematical sort, can be understood in terms of various subspaces, some of which are sets of other subspaces. The structure will not be fully inherited in most cases. The proper space for describing individual human being will have a different structure than does the space for describing communal human being. In fact, the spaces appropriate for describing human mind or

heart or hands will be different from that in which they come together to form an individual human being.

I'll make a major warning about a complexification of the meaning of 'space' in later pages of this book. In the above discussion, spaces mean something like the mathematical abstractions of physical space, perhaps as a function of time. Thus it is that artillery instructors can show graphs of shells following arcs, paths which go up from the howitzer and then go down on top of the target. Spaces can be far more general than this, even so abstract as to be essentially beyond physical imagery, but we build toward an understanding of such abstractions by baby steps, starting first with the path of that artillery shell or with the path of a pop-up during a baseball game—which path is 'calculated' very accurately by certain good athletes.

In modern terms, spaces are more abstract. In the chapter, *Non-Euclidean Geometries* [1], of a classic collection of writings by modern Russian mathematicians, A D Alexandrov tells us:

> The real significance of this [more abstract] point of view [about what spaces are] is that it makes it possible to use the concepts and methods of abstract geometry for the investigation of diverse phenomena. The realm of applicability of geometric concepts and methods is extended immensely in this way. As a result of the concept of space the term 'space' assumes two meanings in science. On the one hand it is the ordinary real space (the universal form of existence of matter), on the other hand it is the 'abstract space,' a collection of homogeneous objects (events, states, etc.) in which spacelike relationships hold. [page 155]

A few pages later, Alexandrov writes:

> By a 'space' we understand in mathematics quite generally an arbitrary collection of homogeneous objects (events, states, functions, figures, values of variables, etc.) between which there are relationships similar to the usual spatial relations (continuity, distance, etc.). Moreover, in regarding a given collection of objects as a space we abstract from properties of these objects except those that are determined by these spacelike relationships in question. These relations determine what we can call

> the structure or 'geometry' of the space. The objects themselves play the role of 'points' of such a space; 'figures' are sets of its 'points'.

Don't worry about the apparent complexity of all of this. It will lead to ways of describing human being, or other forms of complex being, by organizing the knowledge we already have and seeing with greater clarity our existing human possibilities and maybe some new human possibilities as well.

3 A Breather to Introduce God Into His Own Creation

3.1 Trying to Center My Work on God

This entire chapter is a bit of a break from the flow of my arguments but is necessary because this book is part of an effort to build a new understanding of this world and more (all of Creation) which will be suitable for founding a new Christian Civilization—unless it can be used to rebuild Western Civilization, a prospect I consider unlikely.

In any case, my main goal in this section is simply to state clearly that I believe that this universe, a world when seen as morally well-ordered, is part of a Creation. And even for those of us who do accept some divine revelations—the Bible and the traditional Christian Creeds and a little bit more, this Creation is the work of a God that we know primarily through His work as Creator. This is a rational belief which can rendered plausible in the doing, by presenting an understanding of this universe, a world when seen as morally well-ordered, as part of a Creation, a great work of the living God of Jesus Christ.

I'm advocating and presenting what might be labeled (somewhat tongue-in-cheek) an attitude toward human thought and toward Creation, but that 'attitude' is part and parcel of certain ways of human thought, mostly abstract reasoning in the form of 19th century and early 20th century topology and geometry and differential geometry—three greatly overlapping fields of mathematics. To be sure, this material is somewhat outmoded. I'm employing a teaching strategy which makes some compromises. I myself am just learning the better, more 'modern' (by mere decades) methods in topology and geometry and related (that is, nearly all) fields of mathematics [1].

[1] Much is going on in modern mathematics, but it suffices for now to note that a

The reader should remember, and perhaps constantly remind himself, that I'm working in the way of a philosophically oriented mathematical physicist, trying to abstract from empirical reality to get to levels above our concrete, thing-like world. I want to gain a greater and more abstract understanding of created being than is found in empirical investigations of this world. A time-honored goal pursued by the likes of Plato, Aristotle, Avicenna, Albert the Great, Thomas Aquinas, Leibniz, Descartes, and many others. I'm actually returning a bit to Plato's attitudes and methods in trying to understand created being at a more abstract level by drawing directly from mathematics and letting myself be guided, but not rigidly so, by empirical knowledge. Yet, I'm also working in a way consistent with the ways of modern physics and schools of philosophy which closely follow modern physics; those thinkers try to deal directly with empirical knowledge and go looking for specific mathematical tools to deal with particular problems.

Here is something of a mission statement:

> My goal is to be able to think and speak and write of human animals and their communities, of human being in its individual and communal forms, so that these human beings of various types potentially, though very imperfectly so, meet the criteria I've suggested in my past writings for the definition of certain types of complex entities which have narrative context as well as physical being and relationships to various other forms of physical beings. These criteria are unity and coherence and completeness, all within the context of what human being is and can be extrapolated to be. These criteria raise universes to worlds and human animals to human persons. They also raise tribes and families to human communities which can feed into the Body of Christ.

The mathematical thoughts and methods in this work are, and aren't, presented as tools to the purpose stated above. From a larger perspective, one the reader is guided toward, these mathematical thoughts and methods rise to a still greater importance than a mathematician would accord. They are proper ways of speaking of certain fundamental aspects

major part of it is to make it possible to tackle more difficult problems by defining structures underlying topological work and then to use algebraic methods to analyze those structures.

and parts of created being, abstract and concrete. They are abstract created being, or at least as close as we mortals can come to seeing, describing, and understanding such abstract forms of created being. Moreover, I'll even advocate, though not develop, the idea that the Christian idea of one God (manifold) in three Persons ('regions' of that manifold [2]) can be more clearly stated in these terms, dangerously so but 'danger' is a word that applies to all historical developments of the teachings in the traditional Christian creeds (such as the *Nicene Creed*—see `https://en.wikipedia.org/wiki/Nicene_Creed`, the *Apostle's Creed*—see `https://en.wikipedia.org/wiki/Apostles'_Creed`, and the *Athanasian Creed*—see `https://en.wikipedia.org/wiki/Athanasian_Creed`). All of these developments of understandings of the events and revelations recorded in the Christian Bible were eventually accepted as orthodox.

The theological background is important, though theological propositions and arguments for and against those propositions aren't a directly central part of this work. So far as theology goes, what is important is a re-engagement with all of human knowledge. What is even more important is an effort to produce an understanding of this world as part of the Creation described in terms of a different and older theological viewpoint in the book of Genesis.

Over the past few centuries or more, Western Christians have retreated from a proper engagement with God's Creation. Many modern scientists and philosophers now work actively to sequester religious beliefs in the most general sense. Sometimes this is as crude and as juvenile as claiming that the success of the equations of modern physics prove that there is no Creator of this universe, despite the fact that there were some strong, sometimes unconventional, believers among the true geniuses who created modern physics. At least some of the non-believers, such as Einstein who seems to me at times an atheist and at other times a pantheist, didn't claim any categorical knowledge that the God of the Bible didn't exist.

It is the very weaknesses in thought of most of the modern skeptics and non-believers in science that led them to offer battle on the wrong field; unfortunately, far too many theists have accepted the challenge on that field. Not me. I'm not bothering to accept challenges from anyone who thinks that God's existence and power rise or fall with the Big Bang

[2] But also the entirety of that manifold—a complication beyond my current capacity to describe in these mathematical terms.

or with Schrodinger's Equation. See the demanding but richly rewarding *Bangs, Crunches, Whimpers, and Shrieks: Singularities and Acausalities in Relativistic Spacetimes* [7] by the philosopher John Earman for a much deeper discussion of this matter—at the end of the book which is mainly concerned with a technical philosophical analyses of the meaning of possible 'defects' in the spacetime of our universe, he directly engages the efforts to 'prove' God's existence by way of the so-called *Big Bang*:

> Speaking purely personally now, it strikes me as bordering on the sacrilegious to see God's creative force as able to operate only at a singularity or ideal point. It is more to His glory if He operates everywhere and everywhen, and if He operates independently of such contingencies as whether there is an initial singularity and, if so, what type it is. Those who want to find God in the big bang should beware of falling into the trap of relegating God to the diminishing interstices left by modern science. Once the trap is recognized it is easy to escape using God's supernatural attributes. If there is no first instant for the physical universe or no prior physical time to the big bang at which God can operate, no matter. The Creator "may be conceived to exist in a metaphysical time" and "to exist temporally prior to the inception of physical time". [Quotes are from Craig's article *Professor Grünbaum on Creation*, found in the journal *Erkenntnis*, 40, 325-341.] The constraints of physics cannot bind the Creator. But precisely to the extent that a supernatural cause of the beginning of the universe does not have to answer to the constraints of nature, scientists qua scientists are entitled to ignore it.

More succinctly, Earman had asked why anyone would think God to be found in a singularity (such as the Big Bang) more than in a flower.

I would also add to the major quotation above that "Scientists qua scientists" should be consistent and not argue from their scientific authority for or against any beliefs regarding God, including the basic belief about existence or nonexistence. This doesn't mean that scientific knowledge is irrelevant to discussions or debates about philosophical or theological issues: the existence of the Christian God and Plato's metaphysical principles would each have to avoid direct contradiction with empirical truths.

More than that, I'm arguing that theological and philosophical beliefs do enter into our greater understanding of empirical reality and, at least in my worldview and into our greater understanding of any greater reality (all of Creation) of which it is a part.

What we need to do, and what I'm doing, is to step back and take a more global view—as Newton and Einstein did—from various directions, Christian or pantheistic. In this book, I'm taking as the central task that of describing human animals in a way consistent with Christian understandings of human being as individual and community. Mostly, this stepping back is into realms of abstract being as captured in certain mathematical fields which have been well-developed in the past two centuries or so. I'll conjecture that it's not just coincidence that those fields were so well explored by mathematicians and scientists—human thought is moving with the problems and opportunities raised by our responses and lack of responses to our increased knowledge of the empirical world and to the increasing richness and complexity of human being and the increasing richness and complexity of created being in general. This requires also that the scientific and mathematical knowledge of this universe and of its 'contents'—especially human beings—can be seen as consistent with Christian beliefs about human beings as mortal creatures who could share an unending life with the God of Jesus Christ.

Why has there been no adequate response in the form of a richer and more complex understanding of Creation by philosophers and poets and historians? There have been good initial responses—especially by those artists and writers and musicians willing to explore abstract regions of reality. Other responses were off-target, and still others were more in the nature of non-responses. On the whole, we live in cultures which are much like collages: the assembled scraps represent different realms and types of created being. Even the most scientistic of thinkers love but have no way of understanding in even the most speculative of ways what it is that loves or what it is that is loved. Of all the men of history, we modern men know the most but understand the least because we enjoy various types of created being but struggle hard to drive out of our intellects any greater understanding of being, even of human being.

Let me be more specific to the failures and weaknesses of those who share my Christian faith: Why has there been no adequate response in the form of a richer and more complex understanding of Creation by Christians?

In *The Unintended Reformation: How a Religious Revolution Secular-*

ized Society [15], the historian Brad Gregory tells us truly that Christians need to return to Christian practices, ways of life and habits and attitudes and thoughts. Even the most academic of Christian studies will stay on course only when those who study are praying and participating in communal liturgies, reading the Bible and participating in good works as Christians. Professor Gregory also tells us a return to Christian practice and to a stronger faith will be possible only if someone is able to deal with the wide range of modern knowledge to show how we can see our world as a Creation and talk and write about it as a Creation.

Gregory claims, as do I, that many Christians know the attacks upon theistic beliefs have been propagandistic and have not proven, for example, that the very concept of God is meaningless. He knows that pointing this out is no longer sufficient. Too many ignored the danger or even voluntarily took the challenge on the wrong field of battle and the atheists and their allies have won and then moved into the City of Man to take control of schools and mass-media and cultural institutions and political institutions and others.

We need to re-engage on the right battlefield—the entirety of Creation. To do so, we need to ratchet up our thought to a higher level of abstraction that we might consider all being, abstract and concrete, which is within the empirical or speculative reach of the human mind. And, to do so, we need to develop new tools of thought; I'm proposing that we can start by borrowing from certain fields of abstract mathematics. We should back off from the battle on the grounds of physics and biology, recognizing that God the Creator can be seen only in a more total view than that of concrete being and modern, abstract mathematics is likely to be providing us with some tools powerful enough for at least starting this project.

Once we have the proper tools, we can treat this world as part of a far greater Creation, one which includes various levels of created being, abstract and concrete, going back to the fundamental truths which God manifested as the raw stuff of all of created being, the abstract and concrete stuff of Creation. Creation also includes the world of the resurrected, those who will be raised from the grave to share the life of Jesus Christ, the Son of God.

To be sure, the two tasks, of building tools and applying them, need to be done at the same time, as I'm doing in my thinking and writing. In fact, my writings, including this book, constitute a plausible program of the sort Professor Gregory recommends. My program, as it has been developed

publicly on my main weblog, *Acts Of Being* (see `http://loydfueston.com/`), which has not drawn much attention—I do have readers in various countries, mostly the United States and China though I've had surges of downloads in France, Ukraine, and Israel as well as steady downloads of one book, *Four Kinds of Knowledge* [10], in the Philippines and India and various African countries. But I don't know who these readers are.

It's possible I'm building a foundation which is slowly being accepted as a plausible base for a new or revived Christian civilization. It's possible my writings are simply seen as some sort of eccentric but interesting reaction to Western post-Christian ideology. It's also possible my writings are at least inspiring to someone who will build a different, maybe better, foundation for a new Christian civilization—perhaps in a way more to the liking of those from a region not part of the West.

We need to make the world once again safe for Christians and their beliefs, to make the world one in which Christians can pass on their faith to the children of their communities without public schools and publishers and entertainment companies and others interfering to uproot that faith in the minds and hearts and hands of children.

3.2 The Empirical Foundations of Abstract Systems of Thought

All of our knowledge and thought comes from God's thoughts as manifested in Creation, including this universe with its concrete thing-like being. Even the structures and basic elements of our most abstract systems (say those of mathematics and logic) are drawn from empirical reality by exploration and experimentation. I am most certainly not denying that there are valid abstractions, some of which can even be labeled 'absolute truths' (at least within the context of this particular Creation), but I am saying that abstracted Darwinian selection processes are the basis of even the best of human thought, which would mean that this universe and the selection processes it has generated on earth have produced a human brain which can form a mind, a set of abstract relationships, receptive to and consistent with more direct revelations.

In a highly regarded mathematics textbook, *Topology* [18] by Hocking and Young, we can read on page 1:

> In applying the unifying principle of abstraction, we study concrete examples and try to isolate the basic properties upon which the interesting phenomena depend. In the final analysis, of course, the determination of the "correct" properties to be abstracted is largely an experimental process. For instance, although the limit of a sequence of real numbers is a widely used idea, experience has shown that a more basic concept is that of a limit point of a set of real numbers.

Because of this process of experimentation followed by a process of analysis and contemplation—possibly leading to more experimentation, the understanding of the field of 'topology' has changed over time, though each of the understandings has held true even as a new understanding was added.

Topology was first envisioned as an abstraction of geometry; as Euclidean Geometry was the study of the movements of rigid geometric entities (abstracted from reality), so was topology a study of the possibilities of deforming some of those geometric entities into other geometric entities, say, a square into a circle, by the use of transformations having certain well-defined properties such as that of continuity. The standard video easy to find on the Internet shows a cup being deformed 'continuously' into a donut. (See *Topology* at `https://en.wikipedia.org/wiki/Topology` for one example.)

And so it was that topology was partially re-envisioned as the study of 'continuity', the study of 'limit points'.

More recently, topology has been again partially re-envisioned in terms of abstract algebra, abstract relationships and transformations. The above referenced textbook, *Topology* [18], represents a relatively early work in that process, starting gently with what is called limit-point or general topology and then working in algebraic viewpoints and methods.

The main point in this limited context is that this has all come about as a result of exploration and experimentation, not as a result of setting up pure and ideal systems of axioms. Those systems are encapsulations of hard-earned knowledge gained by reasoning about experimental knowledge, including proposed systems of axioms. Such systems of axioms now play a major role in all fields of mathematics and should maybe play a greater role in other fields of human effort, but the useful such systems—those which hold up to hard use and are productive of further fruits—develop in surprisingly empirical ways and stand or fall by way of their usefulness and

fruitfulness. Even light readings in topology will show that the processes of exploration and experimentation has resulted in very abstract definitions and ways of analysis—it's hard for a neophyte to know what some of those definitions can mean and serious mathematicians will simply point out that that particular definitions of 'topological space' or 'limit point' are the well-established results of an effort to, my words, distill out some essence of topology. They work; the essence is for real and more recently distilled essences are more real than earlier distilled essences.

This is pragmatism of a sort, a pragmatism of the best sort, and it has developed in the most abstract of human fields of thought. Perhaps this is what William James and other hardheaded pragmatists were aiming at—not the denial of abstract truths and abstract ways of thought but an insight into the dependence of a flesh-and-blood creature upon its concrete world, a dependence which extends even to abstract thought. In other words, perhaps a truer pragmatism makes no statements about the nature of truth but only about the nature of human efforts to discover truths, efforts which might well be movements from particular or restricted truths toward greater or more abstract truths.

A directed selection process which leads to valid systems of abstract thought? It's certainly a natural process which is selective though it's not natural selection in the purest sense of Darwinism but rather a sort of directed, purpose-driven selection. After all, mathematics and other human activities have actors who are flesh-and-blood as well as rational; as eccentric as Kant or Gödel might have been, they weren't angels or any other sort of disembodied intellects. Human abstract thoughts involve complex relationships which aren't fully embodied but the underlying processes are those of encapsulating those abstract thoughts in networks of neurons and adrenal glands and so forth.

4 Understanding Human Being in Terms of Modern Science

4.1 Overview

We need not only abstractions as tools to model thing-like being. We need to recognize the existence of abstract being and to treat all forms of being. We in the West need to recover our understanding of Creation in its entirety.

Much of higher-level human thought, especially that drawn from the ancient Greeks, has borrowed from mathematics even when dealing with moral issues. I don't know the philosophical and moral etymologies, but you can think of the straight and narrow path as a clear example of borrowing from geometric intuitions, whether from the Greeks or not—I don't know.

In recent centuries and maybe beginning during the Middle Ages, mathematics and science have been developing much faster than other realms of knowledge and perhaps even faster than most artistic fields, but there has been relatively little borrowing of insights and conceptual tools from mathematics and science, though some borrowing of quantitative tools.

Modern sciences, mathematics in its pure form perhaps even more than physics and evolutionary biology, have revealed some very fundamental aspects of being; in Christian terms, modern sciences have revealed some of the thoughts God has manifested as the primary, raw stuff of Creation. Yet, Christians, especially Christian intellectuals and the ecclesiastical leaders, have refused to acknowledge our need to deal with Creation in understanding the Creator. As some Medieval Scholastics taught, including Aquinas: most of what we know about God comes from His effects in Creation. Here is a specific quote from one of the Biblical commentaries of Aquinas:

> [J]ust as a disciple reaches an understanding of the teacher's

> wisdom by the words he hears from him, so man can reach an understanding of God's wisdom by examining the creatures [God] made... [Page 17 of St. Thomas Aquinas' commentary on 1 Corinthians.] [2]

And again:

> [T]he wisdom which attains to God through the things of this world is not the wisdom of this world but the wisdom of God... [page 51 of St. Thomas Aquinas' commentary on 1 Corinthians.] [2]

In other words, if we wish to understand the Creator or Creation (in practical as well as theoretical ways), we must study physics and mathematics and history and biology and chemistry and literature and so on. We must understand the world and all the abstractions from which it was shaped and we achieve this understanding through the various specific sciences. The proper study of mankind is all of Creation. We must study space and time and the abstract relationships of mathematics and the workings of DNA and natural selection and human history and literature under the conscious acknowledgment that all of this is the manifested thoughts of God.

Enough theological justification and motivation for now. Back to qualitative mathematics.

4.2 Topology and Geometry: Moral Shapes

Literate human beings of the modern world, even some who are mathphobic, may know that geometry and topology are the study of shapes, some unchanging, and some which can change into other shapes. Let's examine very informal definitions for these two fields of study:

- Geometry is a branch of mathematics concerned with questions of shape, size, relative position of figures, and the properties of space. [See *Geometry* at `https://en.wikipedia.org/wiki/Geometry`.]

- In mathematics, topology is concerned with the properties of space that are preserved under continuous deformations, such as stretching

and bending, but not tearing or gluing. [See *Topology* at `https://en.wikipedia.org/wiki/Topology`.]

In the article on *Topology* referenced above, they show various figures including an animation of the "continuous deformation" of a cup into a donut. A geometer studies cups and donuts as they are and might say, for example, "A donut is like a car tire but much smaller, but a donut isn't so much like a cup." A topologist studies an entity with one loop, one 'hole', which is deformable by certain allowed processes into another entity with one loop and concludes a donut is indistinguishable from a cup under some important and useful assumptions.

The joke is: a topologist doesn't know the difference between a donut and a cup. Apparently, a topologist might try to drink from a donut or take a bite out of his cup. This isn't just an eccentric idea—topological concepts are used nowadays in physics and chemistry, in engineering and in computer science (even by practical programmers and not just theorists).

The point of much of this book is that we who work in fields of thought outside of physics [1] and engineering can also borrow tools and concepts from abstract mathematics. By doing so, we can introduce a certain appropriate rigor and some freshness into our thoughts about reality, that is—all of Creation. This book is a first step. Not only will I make no efforts to construct actual quantitative models where possible, I'll also not do much to explore a higher level of abstraction than what communicates a need for openness to the borrowing of mathematical results in order to talk about qualitative being or being which is both qualitative and quantitative. Of specific interest is the capability of talking about complex entities, such as human being or even entire universes, as various sorts of wholes which are made from, or can be decomposed into, parts which are substantial entities in themselves.

I have claimed often and in no uncertain terms that human being is communal as well as individual. A human community, or a communal human being, is made of individual human beings in the way of a building being made of bricks and plumbing and wiring, of the architectural plans and the entirety of minds and skills of engineers and carpenters, not in the way of a building being some simplistic pile of bricks and mortar and copper and glass and so on. We could also say that individual human

[1] At times, I use physics as a stand-in for all the fields considered 'science' in modern, limited terms: chemistry, biology, and so on.

beings come together in various complex and historical ways to form true human communities, that is, communities which have a real and not just nominal existence. This second perspective is the proper one in the sense that it agrees with the evolutionary and developmental nature of this world. The problem is that models of those individuals won't easily be pasted together into a smooth entity but we can start learning how to do so by thinking of complex beings as small planar regions being pasted together to make a sphere or other such object. It wouldn't be a very pretty sphere. With purely quantitative models, there are methods for smoothing in one planar region to its neighbors, but that could imply brutalities done to those regions seen as representing or modeling human beings.

I'm writing about 'real-world' mathematics, knowledge and skills which are used to make GPS as accurate as it is, knowledge and skills which are being used to explore longfar agoway regions of this universe. In the first half of the 18th century, a great mathematician named Karl Fredrich Gauss tried to determine the shape of the space of our universe by taking measurements from various mountains in the Alps. He had the right idea but not accurate enough instruments and not a big enough distance. Alternatively, he could have measured some curvature, though not necessarily that corresponding to the universe as an object, if he'd been taking measurements near a black hole. I'll start addressing this sort of knowledge when I deal with manifolds and other, related mathematical objects or concepts.

What does this have to do with the shape of reality, especially of human being in individual and communal forms? We already have "straight-shooters" and "straight and narrow paths" as analogies for honesty and doing what is right. That is, we see truth in straight paths. We see spheres as more perfect than squares which are more perfect than rectangles which are more perfect than irregularly shaped polygons. We see beauty in symmetric things or even ideas symmetric in some way.

But much of our shorthand knowledge for making sense of reality, for making morally good decisions quickly and without large expenditures of energy is no good. We have expanded human being, perhaps even creating new forms of human being. We have done this by exploring empirical realms and then exploring the abstract realms which can be seen in mundane stuff which forms the human body as well as more exotic forms of matter and energy and fields. We discovered how to work with infinity and then discovered there are 'numbers' larger than the infinity of $(1, 2, 3, \ldots)$. We found new ways of telling stories and new ways of making music. Our

human communities have grown very complex indeed and often very large. The genius of Aristotle and Augustine and Darwin never anticipated what we have discovered and what we have developed, nor did the genius of Plato and Newton and Leibniz.

It is somewhat true that we modern men are confused puppies because we walked away from traditional truths, but some—perhaps many—walked away because those truths had, in fact, been revealed as a mixture of absolute truths, provisional truths, partial truths, and even falsehoods. As a result of conflicts between traditional truths (at least as stated) and new knowledge about our world, some chose to go with one and some chose to go with the other. It's not obvious to me that one group or the other were, in any sense, morally superior—except maybe those who stayed with tradition to protect their children and the structures of Western communities. In these somewhat distorted terms, the moral superiority would have been with those who chose to honestly respond to the new knowledge while making an effort to retain—truly retain and not nominally retain—what was good in tradition.

So it is that we have problems with understanding the different ways of being honest and doing what is right in different ages and cultures. In the 21st century, we can no longer even understand ourselves because we've been overwhelmed by knowledge of human beings different from us; our stores of concepts and of words and of grammatical structures aren't up to the task. In the spirit of my effort: we need not ways of understanding men of the same shape who interact in the ways proper to their own culture; we need to understand men of different shape and different culture, to be able to think and write and speak of men who are cups as being the same as men who are donuts—in some important and significant sense of 'same'.

Some human beings are old-fashioned geometers who see a cup as a cup, a donut as a donut, and never the two can be seen as similar. Others have thrown away that old-fashioned knowledge of geometry and are trying to live in an incoherent world of shapeless objects. And others follow Lenin and Stalin in thinking to reshape donuts as cups, using brutal processes if necessary. Still others, including most of the Western power-elite, start by offering peace and prosperity to cups to reshape themselves as donuts but then turn to the brutal means of Lenin and Stalin on those cups which aren't sufficiently cuppish.

One of the takeaways of the above paragraphs is the need for a good way to perceive and understand the big picture. We have so many statistics

that detail, sometimes accurately and truly and other times not, differences between—as a prominent example—Sub-Saharan Africans and Eurasians. Even the most important of those details don't add up to the big picture, that is—a greater understanding of modern human being in its entirety. What's needed is something similar to what I'm trying to make possible in this book.

Yet, abstract knowledge needs to be supplemented by not only that statistical knowledge but also anecdotal knowledge. So it is that I've often told a tale about my bookish inclinations when I was young and then my shocking discovery, in 4th grade, that I'd have to start watching *The Monkees* if I wished to be able to talk to my peers. That was in the phase when Americans enjoyed prosperity and took the bribes to reshape themselves and their children; I apparently was made of a somewhat inflexible material, though I tried hard enough to reshape myself that I lost my way, in a manner of speaking, at college when I couldn't do well by just showing up in class and browsing through the books and then found I didn't know how to study. You have to be shaped properly to study well enough to handle advanced physics and mathematics at a research university. I persevered, did okay after a bad freshman year, and then left academia, fed-up with... Just fed-up, but probably with all of American culture. I didn't want to be stupid and lazy but that's how I was shaped just because I was smart in a town of those who worshiped mediocrity and longed for the plastic worlds of *Father Knows Best* and other early examples of the morally trashy version of the American Way.

We've learned and taught with increasing dogmatism over the recent decades that all human beings are cups, though some might be more cuppish than others. We don't always treat donuts in brutal ways, but it's clear to most that donuts are just defective cups and that is how our institutions have increasingly treated them. We've learned and taught such dangerous and anti-human doctrines in the churches and the synagogues, in the broadcasts of mind-warping and exploitive trash—*Sesame Street* being a prime example of a childish show that prepared the innocent for such stuff as *Rambo* and video-games, but even *The Monkees* with those good-hearted and pleasant and mildly talented young men also scattered the thoughts of boys and girls when they should have been learning how to concentrate, how to eventually carry out difficult tasks by a sustained effort, tasks of the mind and the heart and the hands. Television and movies and popular music in recent decades have mostly developed our primitive

rhythms and our monkey eyes (no pun intended). The interested reader can start looking into this by reading the books of the teacher and educational theorist, Jane Healy. See `http://education.jhu.edu/PD/newhorizons/future/creating_the_future/crfut_healy.cfm` for her own overview of the issues discussed in her very important book: *Endangered Minds: Why Children Don't Think and What We Can Do About It.*

Minds, even more than brains and much more than hearts, can be very different in different human beings, but our educational and cultural leaders have decided that we can all attain human dignity only if we are all the same in most ways. We must reshape the donuts to be cups...

"Oops! These donuts all seem to be just crumbling when we try to reshape them as cups."

We face the same problem in dealing with the civil rights so important to the American way of life and the American way of governing and administering justice. We can't deal with equality before the law and equality in some basic opportunities—such as access to education worthy of the name.

We also consider each child to be a freestanding individual rather than a human being who is an individual and also his communities, especially his family when he first starts school. If that child is very smart but comes from a community not respecting and maybe never having experienced much education, the reshaping process might be partial and might take multiple generations.

We're a people who want it all and want it now, even in our most charitable desires for others. So it is that we'll help those poor donuts if only by crumbling them and then declaring them to be cups.

My claim has been that we've both discovered and helped to create a world too complex to be even described by our traditional concepts and language (words and grammatical structures). This situation has overwhelmed even powerful minds, let alone those who depend upon their communities to supply nearly all their thoughts on more complex matters. More surprisingly, it doesn't seem to have called forth much of an effort by those with minds both powerful and creative; the best that can be seen are some efforts by science fiction authors such as Harlan Ellison and Phillip Dick which are suggestive more than profoundly insightful.

It would seem logical that we should begin exploring these richer and more complex domains of modern science and mathematics, domains which have been growing and developing very rapidly; we explore in order to borrow that we might speak of differently shaped human individuals and differ-

ently shaped human communities. But it is a borrowing back. We human beings began speaking descriptively of what lies around us and speaking of procedures for starting fires. Even theoretical physics has had to borrow the qualitative words and grammars useful for purposes which were often outright 'anthropomorphic'. This borrowing of terms such as 'mass' and 'weight' has caused endless problems for those teaching or learning basic physics where the two terms refer to different phenomena—though appearing the same so long as you know about gravity by its effects at one particular strength.

Let's return to topology and strengthen our understanding a little. Topology isn't really, rather—isn't mostly, the study of how to continuously deform a cup into a donut; we're closer to the truth when we claim topology is the study of sets or objects within well-defined topological spaces which continuously deform one into another after allowing for the proper invariants. Cups and donuts both have a loop which can't be eliminated in three-dimensional space by continuous deformation. So topologists study the continuous deformation of a three-dimensional space (or parts of that space) into another three-dimensional space when a loop is present in both spaces. They study deformations a lot more general than that, but that is the truth behind the topologist who drinks from a donut and eats a cup. He's living in a space with only one loop which can't be eliminated, though he'll return to a far more complicated space when he goes home to his wife and three children. I'll try to produce many hints of what I'm really up to and this is one: to restore unity to our worldview, our understanding of all of reality, we need conceptual tools which provide bridges rather than barriers, which allow us to understand there is a conceptual level, above 'mathematics' as most understand the term, where holes in our lives are like that hole in a cup and that hole in a donut.

So, it is better to say:

> Topology may be considered as an abstract study of the limit-point concept. (See page 1 of *Topology* [18].)

I've used the term 'limit-point' a couple of times without defining it. Here goes:

Limit point In mathematics, a limit point of a set S in a topological space X is a point x (which is in X, but not necessarily in S) that can be

> "approximated" by points of S in the sense that every neighbourhood of x with respect to the topology on X also contains a point of S other than x itself. Note that x does not have to be an element of S. This concept profitably generalizes the notion of a limit and is the underpinning of concepts such as closed set and topological closure. Indeed, a set is closed if and only if it contains all of its limit points, and the topological closure operation can be thought of as an operation that enriches a set by uniting it with its limit points. [See `https://en.wikipedia.org/wiki/Limit_point`.]

Note that all points are limit points in a continuous set of the same sort as the closed interval from 0 to 1 on the real-number line. The points are also all limit points in the open interval from 0 to 1 on that line. Cups and donuts, as ideal objects in a topological space, are continuous and so all points inside and on the surface of those objects are limit points. That the points on the surface are limit points tells us it is closed—other sorts of materials studied by scientists and engineers, and used in our technology, might be open but the atoms on the surface, approximated as points of a continuous mass, would also be limit points.

Thus it is that the topological reshaping of a cup into a donut takes each and very point of that cup into a point of the donut, without any tearing or gluing of either object during the process.

Hocking and Young tell us that:

> The set S *has a topology* (or *is topologized*) provided that, for every point p in S and every subset X of S, the question "Is p a limit point of X?" can be answered. (See page 1 of *Topology* [18].)

In that spirit, I'll expand a little on the idea of deforming a cup into a donut. Suppose we have a, speaking roughly, well-behaved function $f()$ which models a deformation of a cup into a donut. Suppose further it has an inverse $f^{-}()$ which models a deformation of a donut into a cup. So, if we claim a cup deforms into a donut, we are claiming:

> If (x_1, x_2, x_3) represents a limit point of a region in a cup, then $(y_1, y_2, y_3) = f(x_1, x_2, x_3)$ is a limit point of the corresponding region in a donut.

To beat a dead topology, let's claim a donut deforms into a cup. This is to claim:

> If (y_1, y_2, y_3) represents a limit point of a region in a donut, then $(x_1, x_2, x_3) = f^{-1}(y_1, y_2, y_3)$ is a limit point of the corresponding region in a cup.

Think of this in general terms. That is, in fact, the spirit of much abstract mathematics, including topology. Build up theories and fields of thought based on intuition and empirical knowledge and then abstract, abstract, abstract. It's possible that this process has reached a high point for now, a tide reaching a high point on a beach though higher points might be reachable at some time. This possibility of a high point comes just because the extremely high level of abstraction has gone far, far ahead of general human thought. This would seem an unstable situation to be fixed by some time spent bringing some of the power of abstract thought in mathematics and mathematical physics and other fields—including the 'practical' field of computer science—into the general stream of human thought [2].

There is a minor problem in my above discussions of topology, but one worth worrying over just a little.

"A topology."

"The topology."

Spaces have topologies and objects, or at least their shapes, have a particular topology when seen as being in a space—assuming that space has a topology, assuming that space has been topologized. For example, we live in a 'thin' region, the surface of Earth, which can be idealized into a topology described as a 2-manifold—the topology of a 2-dimensional sphere or surface of a 3-dimensional solid ball and also the topology of any surface which can be continuously deformed into a 2-dimensional sphere in the way a cup is continuously deformed into a donut.

But topology, as a mathematical field, is the study of specific topologies. Keeping this important nit-pick in mind, the context should make it readily certain which meaning is intended.

> Those factors that dictate the choice of a topology for a given set S should become more apparent as we progress. In many

[2] Of course, you could lose a lot of money betting on trends continuing or changing direction in intellectual fields, in the economy, in voting allegiances, and many other matters.

> cases, a "natural" topology exists, a topology agreeing with our intuitive idea of what a limit point should be. ... In general, however, we require only a structure within the set S which will define **limit point** in a simple manner and in such a way that certain basic relations concerning limit points are maintained. (See page 2 of *Topology* [18].)

It might be worthwhile to reread the above quote several times or even get hold of a copy of the source, *Topology* [18], and read the early pages (or more). These words describe the attitude and a bit of the method for the abstraction of mathematical knowledge into the form where the ideas can be borrowed, even 'stolen', and used for more general understanding of human being and of all sorts of created being. For those who haven't studied the underlying mathematics, at least not in recent years: the quotation is pointing toward the expansion into more general realms of mathematical knowledge gained in analyzing 'geometric' objects; the concepts of limits on the number line ($\Re$) or the Cartesian plane ($\Re^2$) can be extended to limits of abstract sets of objects which might not be numbers, perhaps functions or other mathematical objects or even non-mathematical objects. (See *Analytic geometry* at `https://en.wikipedia.org/wiki/Analytic_geometry` for a discussion of the Cartesian plane. See *Rene Descartes* at `https://en.wikipedia.org/wiki/Rene_Descartes` for a discussion of the thought behind Descartes' development of analytic geometry, thought which was something of a unity of theological and philosophical and mathematical thought.)

To get some intuition about non-mathematical sets and possible limits, think of a set of species (each itself a set) of hominids with some admittedly biased understanding of anatomically modern hominids who are us as the current human state closest to the limit which is 'true' human being. This is the limit in a complex space of states where each state contains both quantitative and qualitative parts and aspects, concrete and abstract parts and aspects.

4.3 Concrete Human Being

We speak of human being as if it referred, in some sense, to a coherent concept but we never define that sense. After all, there seems to be quite

a bit of difference between various peoples of humanity. There seems to be no currently accepted understandings of human being which make sense of individuals and communities at the same time. As a hint of what is coming: it seems to me possible to take individuals as local regions in a space defined by biologically-based characteristics and to take communities as global or as larger regions with more abstract characteristics, combining individuals so that those larger regions take on some global characteristics. If we take human communities as pre-existing sets, we can work from those sets toward an understanding of the 'points' or the members of those sets. Eventually, there will be a need to develop an understanding of communities as abstract being of a dynamic sort, true being from which concrete entities are shaped. That's graduate school stuff and this book is meant to be a freshman text.

One possible objection from modern liberals of libertarian or classical liberal or progressivist schools of thought is:

> Individuals are what truly exist and communities are just voluntary gatherings or other associations of those individuals. All human relationships, whether or not you call them communal, follow the form of revocable contracts. This leaves fields of moral and political and cultural relationships flat and not curved in any way as to give the communal regions any true identity.

I'll deal with the claims of modern liberals—not any political or social or cultural claims but rather the basic principle that human beings are free-standing individuals who are relatively homogeneous in the characteristics which define humanity. That response to modern liberals will be in Part III, *The Totality of Human Being: Individual and Communal.*

For now, I'll be dealing with the variation in human being which we find between groups of humans who have adapted to different environments I'll start with some short descriptions of a few peoples who differ in interesting ways from each other and from even the peoples who live near them:

- There are the peoples of tropical Africa, such as the *Maasai* who seem to have adapted to their environment by becoming tall and slender, increasing the ratio of heat-emitting skin to total body mass. (See `https://en.wikipedia.org/wiki/Maasai_people`.)

- There are the arctic peoples, such as the *Eskimo* "who have traditionally inhabited the northern circumpolar region from eastern Siberia (Russia), across Alaska (United States), Canada, and Greenland." These people are typically stocky and have a low ratio of heat-emitting skin to total body mass, even though they wear heavy clothing over most of their bodies. (See `https://en.wikipedia.org/wiki/Eskimo`.)

- There are the *Basque* who are genetically isolated from other Europeans and seemingly a mixture of neolithic farming peoples with early hunter/gatherers, who "primarily inhabit an area traditionally known as the Basque Country (Basque: Euskal Herria), a region that is located around the western end of the Pyrenees on the coast of the Bay of Biscay and straddles parts of north-central Spain and south-western France." (See `https://en.wikipedia.org/wiki/Basques` and, yet, if you look up information on Irish Gaels, you'll find that they are tied, by the latest research and opinions, along with the Welsh and some Englishmen and northern Portuguese and some Spanish to the Basque. The ties are through male lines of descent. The historical-geneticists are getting close to a stable and reliable narrative of European ancestry but they're not there yet.)

- There are the *Tibetans*, who are phenotypically mixed and "although Tibetans living at high altitudes have no more oxygen in their blood than other people, they have ten times more nitric oxide and double the forearm blood flow of low-altitude dwellers. Nitric oxide causes dilation of blood vessels allowing blood to flow more freely to the extremities and aids the release of oxygen to tissues. This and other advantages in physiological function at high altitudes have been attributed to a mutation in the EPAS1 gene among Tibetans."

These and many other peoples cover much of the earth. They have different physical attributes (tall and slender or short and sturdy or medium in height and lean or other) adapted to different environments (arctic or tropical plains and deserts, valleys and plains of temperate-zone Europe or elsewhere). They speak different languages which have descended from 'proto-' languages along with many others in various families—some of which originate in times too old for linguists to draw the connections. They have

followed different cultural paths and have developed different religions and political systems.

We human beings are not all the same but we are much alike; we are a species but we have many varieties with varying characteristics of a superficial or deep sort. We haven't done so much to clear up the many confusions or to bring to a greater state of coherence the many insights. This is one of similar sorts of problems which arise when we try to understand the world around us. My proposal is to think in terms I've suggested in many of my writings [13]: the concrete, thing-like being of our world is shaped from more abstract forms of being. If we look at the things of this world in terms of the proper abstractions, we can see the relationships. This is clearly an existing perspective of science when operating in a descriptive mode, but I'm proposing that abstractions are actual being, a sort-of Platonism in which the Reals aren't perfect versions of complex things but rather the building blocks, relationships and stuff, of those things as well as varying characteristics which can become part of concrete things, simple or complex.

As is true of donuts and coffee cups, Maasai and elephants, Eskimos and polar bears, Sioux and bison, aren't the same but have similarities which allow us to classify all of them as mammals of specific species: *Homo sapiens sapiens* and *Loxodonta africana* [3], *Homo sapiens sapiens* and *Ursus maritimus*, *Homo sapiens sapiens* and *Bison bison*. Similarly, when we speak of peoples, we can classify them, in cultural and linguistic and genetic terms, as being Maasai or Eskimos or Sioux. There is great capacity for adapting in various ways, culturally and linguistically and genetically and others, but these peoples can't easily live in each others environments and can't just be turned into each other. Even if we tried to conduct some vast evolutionary and historical experiment by repopulating various environments with one type of people, the results would be different. Thousands of years of suffering and then perhaps prospering by Eskimos placed in Kenya would result—given at least some survivors of such a cruel experiment—in a people very similar to the Maasai in some ways important to humans living in that environment and, by chance, very similar and very different in contingent matters not so greatly constrained by the environment.

So, how can we speak in a more coherent and more insightful way of the

[3]This is one of two species of African elephants, the larger species found in bush country rather than in the forests.

various peoples of humanity, of various species of animals, of the environments of various planets. If human beings adapt well to the environments of other planets, will they be different enough from Earthlings as to be a separate species? What would be the moral ties between those two groups, separate species or not? What obligations would they have to each other?

How about those hunter-gatherer peoples of Africa (Central African pygmies, South African bushmen, and a few others) with total population in the hundreds of thousands who split from other anatomically modern human beings about 250,000 years ago, 100,000 years before that false Mother-of-us-all, Mitochondrial Eve? Since that mystery woman was the mother of all true human beings, then are those pygmies and bushmen not really human beings? Or are we just damned, self-righteous, high self-esteem idiots? Or do we simply tend to create future problems by being sloppy in our thinking and in our use of theories and contingent facts?

I don't know any of those answers at this point, but I suspect I could ask those questions in a way where they make sense, specific sense subject to rational discussion not much like the mushy discussions we modern men hold about our ties with each other. I've stepped back in this book; I'm trying to provide some useful tools abstracted from the powerful tools of modern mathematics. These tools are descriptive and might well lead to the development of more powerful sorts of tools of analysis.

I can say this much: In some sense, we human beings are all the same. I can also say: In many senses, we human beings are different as individuals and as peoples adapted to different environments by way of contingent processes.

For now, I'll conjecture that there is one level of abstraction where all human beings are shaped the same, though we don't all have the bodily shape of a Olympic champion or that of a conqueror of Mt Everest, the mental configuration of a Nobel Laureate in physics or that of a composer of symphonies, or the organizing skills of an entrepreneur or those of a statesman. I think most of us know this, at least those who are cognitively and morally sane.

And, again, I'm trying to provide ways to speak and think about this subject, ways which might allow us to advance in our understandings of human being and of other complex forms of being.

Let's travel further into very strange and dangerous regions. I've discussed the various prehistories of Indo-European and First-Farmer peoples. These prehistories consider a variety of information sources, archaeological

and linguistic and even mythical—subjected to the firmest of all sources, analyses of ancient genes and comparison of those genes to the genes of modern human beings. An important book, *The 10,000 Year Explosion: How Civilization Accelerated Human Evolution* [5] by Gregory Cochran and Henry Harpending, tells of the rapid increase of IQs in these groups. Separately, historians of prior generations had noted that a certain type of thinking, the abstract thinking of great concern in this book, appeared in the record of human achievements about the sixth and seventh centuries BC, almost as if brought about instantly by historical and evolutionary standards. These mysterious events occurred among Eurasian peoples, and perhaps a 1,000 years or so later among the Amerindian peoples. They seemed to have occurred in synch with waves of increasing complexity in human communities and in human projects of raiding, disciplined military invasions, and mass movements of large groups.

This is very strange. So much that is good and moral in human life, the technology and complex communities which make it possible for so many to survive and often prosper even when they're sickly or crippled or quite limited in intelligence or other important human characteristics, so much that we rightfully desire for our own selves and for those we love and, more weakly so, for all human beings...

So much that is good, so much, so much, might exist because of selection of intelligence for such purposes as successful raiding, stealing of the food resources or land or even children or women of others. To be sure, it's more complex than that, but it's at least that complex.

4.4 The Amoral Roots of Moral Order

Or...

Are they? That is, are the roots really amoral? More appropriately, do we humans see wrongly when we separate moral order from other sorts of order, raising moral order to angelic, ethereal realms? Once upon a time, when the West was truly Christian, we Western men did that with human nature. By tying Christian beliefs into these confused beliefs, we sought to protect "human dignity" from any hints we're cousins to chimpanzees, more distant cousins to bears, still more distant cousins to alligators and eagles and sharks and so on. We paid a price: when we try to see ourselves as not quite creatures of this world, by trying to see the world of the resurrected

as a realm of pure spirits rather than a completed and perfected version of this world, we badly damage the credibility of revealed truths—directly and indirectly by ceding our central role in a civilization which then advanced by leaving behind its Christian foundations. That the West has no other foundations and is now something of a low-flying castle in the sky is largely the fault of Christians who failed to see God's Creation as it truly is rather than as we would wish it to be for our comfort and convenience.

I've dealt with these issues, but I risk repeating earlier thoughts because of the problems caused by both Western Christian leaders and the so-called power-elites of the West as they thrash about, trying to save their own power and wealth—even at the risk of destroying what is left of the West, what could be used as part of the foundation for reforming or rebuilding a once great civilization which promised even more greatness. Powerholders in the Vatican and other powerholders in Western Christianity have joined with the various sorts of neo-cluelessness in idealizing morality and human communal structures. In this way, they seek to hold onto power they no longer are capable of even understanding, let alone using properly.

This incoherent behavior of those who hold power in the West comes from a strange way of thinking among many in the West over recent centuries, even some who were extremely intelligent. In the spirit of the times, I could claim those many in the West, including most Christian leaders and intellectuals, apparently acquired their understanding of order and moral order from some book titled: Order and Moral Order for the Totally Clueless Who Wish to Remain Totally Clueless.

Life didn't jump from an oceanic swarm of eat-or-be-eaten critters up to Francis of Assisi, George Washington, and Albert Einstein. It also didn't jump from the common ancestor of humans and chimpanzees to human beings of such high achievements and (mostly) good moral order. It didn't jump from early hominids to Hammurabi, let alone to da Vinci or Goethe.

Civilized human life partly developed by way of barbarian warlords who could organize military raids and handle the logistics of taking captives to the slave-markets, by way of similar men building irrigation systems with labor extracted from human beings without a hint of representation, by way of the brutal conquests of the Chinese emperors as well as those of Mongols and Manchus and by way of decentralized conquest of European lands by Indo-European warlords...

Humans can only develop into civilized human beings, their tribes into civilizations, if they have the proper characteristics. The family lines of

those particular humans gain those characteristics by way of natural selection, including selection for good social characteristics in the cases of human beings, voles and some other rodents, elephants, whales, and so on.

There is a nasty matter which arises just because the political theorists and practitioners of recent centuries, non-Christian and anti-Christian as well as Christian, have ignored the realities of evolution in positing that the abstract reasoning abilities of all peoples and of all individuals in those peoples have risen in unison. It's likely that many peoples are missing either the abstract reasoning ability or the proper emotional settings to be properly disturbed by, say, brutal war crimes committed by the leaders of their tribe or kingdom or empire or state.

In general, it takes a high level of reasoning ability to understand the huge and complex human communities of the modern world, to be even potentially self-governing in a meaningful way rather than just being cattle to be manipulated by political machines or demagogues.

Geneticists tell us that a high percentage of Y-chromosomes in Western Europe, China, and probably most regions of civilized order came from a small number of men who were almost certainly great military and political leaders, though I'd guess that many blacksmiths and horse-breeders and others who had knowledge and skills important to the warriors also left a fair number of descendants. At the same time, political and military power seems to have given reproductive advantages to male-lines over many generations, so that the paternal lines of even successful merchants and technologists might have dwindled; many of those other men might have left us most of their genetic heritage through their daughters.

It is hardly surprising that the order of Indo-European warlords established the foundations of what would become Christian Europe. It is hardly surprising that the real but incomplete morality of warrior-bands is still with us in the West. It is hardly surprising that modern attempts to idealize morality as something above and beyond this Einsteinian and Darwinian universe is entangled with forces of moral breakdown, that is, idealists threaten the real foundations of higher morality and that could lead to a breakdown of higher morality.

The rise in IQs across Eurasia occurred rapidly over the past 10,000 years in apparent synch with the development of more advanced stone tools and weapons and then the development of metal technology, of semi-nomadic agriculture leading to true agriculture, of organized warfare and also the organized movements of somewhat larger groups of peoples—logistically

complex movements since women and children and elderly men were in the migrating groups. All of this seems to have also been entangled with the development of harsh forms of order imposed by warriors, including the organization of larger-scale agricultural operations with the likelihood that clueless subsistence farmers were rounded up with threats of spears so that they could dig the irrigation ditches and settle into regions under the control of one warlord or another—the ancient works of Hebrew scripture give us stories of such complex events and make us aware that much suffering came with the benefits of urban living with its technologies.

It's far from clear that modern Western forms of liberal social organization and their associated forms of liberal morality will last. It's far from clear they are the best of currently possible ways to organize our human communities and to develop our systems of morality with rights and duties.

It's quite possible that we of the modern West have misunderstood the nature of order and of moral order so that we have simply exploited the material wealth and ordered communities we inherited, turning those communities into unstable forms so that the next few generations will inherit a civilization in ruins.

We've misunderstood largely as a result of our often self-righteous desire to be kind to all, to rightly accept as human (at least if born) any creature which might be human, but we've made basic mistakes which might lead to horrors at least as great as those of the Nazi Era in Germany. Nature bites back and she bites back particularly hard when an allegedly rational species uses its rationality to do good in unwise ways. And we Christians have to remember that 'nature' is a pointer to various creatures of the God of Jesus Christ.

I've not settled much at all. In fact, I hope I've usefully upset those who are complacent, self-righteous, full of self-esteem. I hope I've upset those who are making themselves feel good at the expense of future generations.

With that, I'll return to some mathematics of a sort.

4.5 Differential Geometry: The One and the Many

I've already been forced to prematurely mention manifold. I'll take care of that glitch which is often forced by the explication and learning of this

sort of material—if you had to know all the prerequisites, you'd never be able to get started. For trained mathematicians or physicists or, indeed, any trained scholar or musical performer or other skilled worker, this is no problem. Over time, the teachers and accreditation agencies in many of these fields have learned what is best learned in the freshman year of college or the first year of apprenticeship in a skilled trade, best learned so that the more advanced learning can take place in a reasonably smooth manner. Obviously, I write here of those capable of self-learning, whether a historian or plumber. Such a restriction bothers me not at all.

Before we can get to the definition of manifold, it is perhaps best to describe a major branch of modern, abstract mathematics:

Riemannian geometry Riemannian geometry is the branch of differential geometry that studies Riemannian manifolds, smooth manifolds with a Riemannian metric, i.e. with an inner product on the tangent space at each point that varies smoothly from point to point. This gives, in particular, local notions of angle, length of curves, surface area, and volume. From those some other global quantities can be derived by integrating local contributions. [See *Riemannian geometry* at `https://en.wikipedia.org/wiki/Riemannian_geometry`.]

The introduction to the article on *Riemannian geometry* tells us:

> Riemannian geometry originated with the vision of Bernhard Riemann expressed in his inaugurational lecture *Ueber die Hypothesen, welche der Geometrie zu Grunde liegen* (*On the Hypotheses which lie at the Bases of Geometry*). It is a very broad and abstract generalization of the differential geometry of surfaces in $\Re^3$. Development of Riemannian geometry resulted in synthesis of diverse results concerning the geometry of surfaces and the behavior of geodesics on them, with techniques that can be applied to the study of differentiable manifolds of higher dimensions. It enabled Einstein's general relativity theory, made profound impact on group theory and representation theory, as well as analysis, and spurred the development of algebraic and differential topology. [See *Riemannian geometry* at `https://en.wikipedia.org/wiki/Riemannian_geometry`.]

And this is a short and sweet definition of manifold:

Manifold In mathematics, a manifold is a topological space that resembles Euclidean space near each point. More precisely, each point of an n-dimensional manifold has a neighbourhood that is homeomorphic to the Euclidean space of dimension n [$\Re^n$]. [See *Manifold* at `https://en.wikipedia.org/wiki/Manifold`.]

The article goes on to tell us:

> One-dimensional manifolds include lines and circles, but not figure eights (because they have singularities called crossing points). Two-dimensional manifolds are also called surfaces. Examples include the plane, the sphere, and the torus, which can all be embedded (formed without self-intersections) in three dimensional real space, but also the Klein bottle and real projective plane which cannot.
>
> Although a manifold resembles Euclidean space near each point, globally it may not. For example, the surface of the sphere is not a Euclidean space, but in a region it can be charted by means of map projections of the region into the Euclidean plane (in the context of manifolds they are called charts). When a region appears in two neighbouring charts, the two representations do not coincide exactly and a transformation is needed to pass from one to the other, called a transition map.
>
> The concept of a manifold is central to many parts of geometry and modern mathematical physics because it allows more complicated structures to be described and understood in terms of the relatively well-understood properties of Euclidean space. Manifolds naturally arise as solution sets of systems of equations and as graphs of functions. [See *Manifold* at `https://en.wikipedia.org/wiki/Manifold`.]

What does it mean to be homeomorphic?

Homeomorphism In the mathematical field of topology, a homeomorphism or topological isomorphism or bi continuous function is a continuous function between topological spaces that has a continuous inverse function. Homeomorphisms are the isomorphisms in the category of topological spaces—that is, they are the mappings that preserve all the topological properties of a given space. Two spaces with

> a homeomorphism between them are called homeomorphic, and from a topological viewpoint they are the same. [See *Homeomorphism* at `https://en.wikipedia.org/wiki/Homeomorphism`.]

In the next paragraph of that article, we can read:

> Roughly speaking, a topological space is a geometric object, and the homeomorphism is a continuous stretching and bending of the object into a new shape. Thus, a square and a circle are homeomorphic to each other, but a sphere and a torus are not. ...[A] sufficiently pliable donut could be reshaped to the form of a coffee cup by creating a dimple and progressively enlarging it, while preserving the donut hole in a cup's handle.

In fact, there is a dynamic image at that page of a cup turning into a donut (torus) and into a cup and so on.

If you don't quite understand the meaning of homeomorphic, don't worry for now, though, in very general terms, it simply means that a small enough region of the earth's surface (a 2-dimensional manifold which is the surface of a a 3-dimensional and solid ball [4]) can be accurately mapped to, or displayed on, an ordinary map—flat and rectangular. When you look for a street on a map of your hometown, you don't even need to think about the distortion of mapping that curved surface area to a flat map. But homeomorphic also means there is a well-behaved reverse mapping from that street map to a global representation of the earth's surface or at least a representation of a large part of that surface. And it remains true that larger areas of the 2-dimensional surface of a 3-dimensional ball aren't going to be mappable in this nice way—the entire surface remains subject to distortions if we map it to a flat representation. In the article on *Mercator projection*—which is not a homeomorphism, we learn:

> Although the linear scale is equal in all directions around any point, thus preserving the angles and the shapes of small objects (which makes the projection conformal), the Mercator projection distorts the size of objects as the latitude increases from the

[4] But always remember that the 'solid' n-dimensional object and even the entire n-dimensional space need not, and usually does not, exist for the (n-1)-dimensional object and 'space' to exist.

> Equator to the poles, where the scale becomes infinite. So, for example, landmasses such as Greenland and Antarctica appear much larger than they actually are relative to land masses near the equator, such as Central Africa.

All such mappings from the manifold to a Euclidean plane, not just the Mercator projection but even those which are homeomorphisms when restricted to small regions, will be distorted in one or more regions if those mappings are used for large regions or even the entirety of the sphere or other surface. It's likely that there will be regions of any global representations of human being (communities) which will have similar problems. For example, the human being of a "great man," such as Napoleon, will be distorted as we map his large region of action and influence, as will be the communal being around him. It will be distorted in reality, in terms of real human being, and not just in terms of the mathematical or conceptual representation of human being—this should give us hints of the reasons that human beings of great power are often constrained by that power, turned and twisted in their own being as they stretch over too large a region for their own good.

The abstract mathematics described in such statements as those in the preceding paragraph are, in fact, forms of abstract being which flow into the concrete human being of Napoleon and of France in that revolutionary period. Created being is created being is. . . ; the concrete, thing-like being of this universe is shaped from various sorts of more abstract being, including some very abstract sorts of being which are explored by mathematicians.

As one result, concentrations of human being (such as the 'liberated' masses of Revolutionary France) and particularly intense contexts (such as those around Napoleon or other "great men") will create greater curvature than that found around an obscure thinker sitting at his computer as he tries to develop new ways of dealing with human being in a world of "concentrations" and of "intense contexts." This points to the still worse situations created when the most mediocre and even most incompetent of men and women reach for the greatness which destroyed Napoleon, a man of some true greatness if he'd only kept some sense of proportion, an attitude of humility.

That Wikipedia article, *Manifold* at `https://en.wikipedia.org/wiki/Manifold`, goes on to tell us about the importance of manifolds to mathematicians and physicists:

> The concept of a manifold is central to many parts of geometry and modern mathematical physics because it allows more complicated structures to be described and understood in terms of the relatively well-understood properties of Euclidean space.

To risk being annoyingly repetitious, a Euclidean space can be thought of as the space including those flat maps of towns or battlefields and even including the maps of possible combinations of height and weight for human males. (See figure 5.1 below.) These maps are of small regions on a global surface which isn't necessarily a sphere and isn't necessarily 2-dimensional though both of those are easy for us to imagine and to learn how to deal with large dimensional spaces and the complex abstract objects those spaces might include.

Roughly speaking, manifolds are studied in differential geometry, though by way of tools from linear algebra and perhaps other fields of algebra and tools from topology; manifolds can themselves be used as a setting for 'modernized' versions of calculus or analysis. When the small regions are the focus, we can call our work 'differential geometry in the small'; when the total surface or some significant portion of it are the focus, we can call it 'differential geometry in the large' or 'global analysis' or something of the sort. That raises the question: What is differential geometry?

The Wikipedia article, *Differential geometry* begins with these paragraphs:

> Differential geometry is a mathematical discipline that uses the techniques of differential calculus, integral calculus, linear algebra and multilinear algebra to study problems in geometry. The theory of plane and space curves and surfaces in the three-dimensional Euclidean space formed the basis for development of differential geometry during the 18th century and the 19th century.
>
> Since the late 19th century, differential geometry has grown into a field concerned more generally with the geometric structures on differentiable manifolds. Differential geometry is closely related to differential topology and the geometric aspects of the theory of differential equations. The differential geometry of surfaces captures many of the key ideas and techniques char-

acteristic of this field. [See `https://en.wikipedia.org/wiki/Differential_geometry`.]

The above description of differential geometry makes it sound highly technical and complex, something for real math-minds. It can be very difficult, as is true of all tough fields of study, but I wish to draw from differential geometry some ideas I've proposed, usually in vague terms:

- Even the most concrete of entities can be composed, in some sense, of entities smaller, in some sense; the greater entity is a real being and so are each of the smaller entities.

- In some cases of serious interests, parts or aspects of those smaller entities can be mapped to a Euclidean space of real numbers: E^n or $\Re^n$ where n is the dimension of the space and $\Re$ is, as always, the real number line.

- In other cases, we might not be able to use the simple (by mathematical standards) calculations and analyses possible in Euclidean space and will be forced to use more esoteric and qualitative entities, sets or collections, and so on.

- In some cases, the whole is—to some serious extent—a larger version of each of the smaller entities. This is called self-similarity. St Paul spoke of the Body of Christ being similar to the individuals which compose it, one perfect Man made of many men. At the same time, I've written of the Body of Christ as being a completed and perfected civilization, the fulfillment of all legitimate human thoughts and desires and activities.

4.6 Taking a Breather

Whew!

But don't worry. The above is something of a conceptual overview of a substantial undergraduate course of studies in mathematics. It can also form a core for the study of complex structures in this world. I'll approach from that direction, emphasizing—or at least hinting at—the usefulness and truthfulness of this material in a modern Christian study of human being,

that is, the study of the Body of Christ in its many individuals and in its wholeness.

I'll give a very general preview of what I will provide in a more detailed, but still quite preliminary form:

> A human being can be usefully and truthfully described as a manifold, but one mixed of parts and aspects; some simple and quantitative in a straightforward manner (a man is measurably taller or smarter in certain ways than another man) so that they can be directly mapped to some Euclidean space, E^n; perhaps some are quantitative in a more complex way requiring directly measurable characteristics to be run through a function, $f()$, defined upon some Euclidean space, E^n; and some are not so quantitative, perhaps having no way to reliably measure a 'distance' between the characteristics of two men, for example, honesty or loyalty.

I wrote the above in a deliberately vague way that might be confusing and most certainly won't be very enlightening to one who doesn't know how mathematicians understand such concepts as 'measure' or 'distance between characteristics'. Don't worry and take things in a rough and intuitive way. I'll explain more later, but not all of it; for now, it's necessary to hint at what's coming so that the reader can prepare to learn what's coming.

Thinking analogical to the thinking in topology and various fields of geometry (especially the field called "differential geometry") will allow us to think, speak, and write of complex entities which can be regarded as coherent and unified and complete (in some strong sense to be explained) though being quite abstract as well, maybe sets of sets which are from different 'realms'.

5 States of Realized or Concrete Being

5.1 Quantitative States

Let's consider something called a 'phase space':

> In mathematics and physics, a phase space of a dynamical system is a space in which all possible states of a system are represented, with each possible state of the system corresponding to one unique point in the phase space. For mechanical systems, the phase space usually consists of all possible values of position and momentum variables. The concept of phase space was developed in the late 19th century by Ludwig Boltzmann, Henri Poincaré, and Willard Gibbs. [See *Phase space* at `https://en.wikipedia.org/wiki/Phase_space`.]

This is food for thought though it may or may not be quite the right way to consider the problem of describing complex entities which can be seen as sets of characteristics of various sorts. So...

What is a space of all possible states? I'll reduce the idea a little and eliminate any explicit dynamical aspects to the system. To differentiate from the use by physicists, I'll use a non-standard term which smacks of metaphysics: states of being. This will prove to be a useful and good nomenclature, conducive to a useful and good way of thinking.

A simple example can be made using the height and weight characteristics of human males and limit the range of such characteristics to the 99.9%, eliminating consideration of those with genetic disorders of the hormonal systems and eating disorders and such problems. I'll make the ad-hoc assumption that those 99.9% fall in the range of 58 inches to 80 inches and

100 pounds to 350 pounds. The following simple graph shows a simple presentation of the likely states of being with respect to height and weight; the numbers come from my imagination.

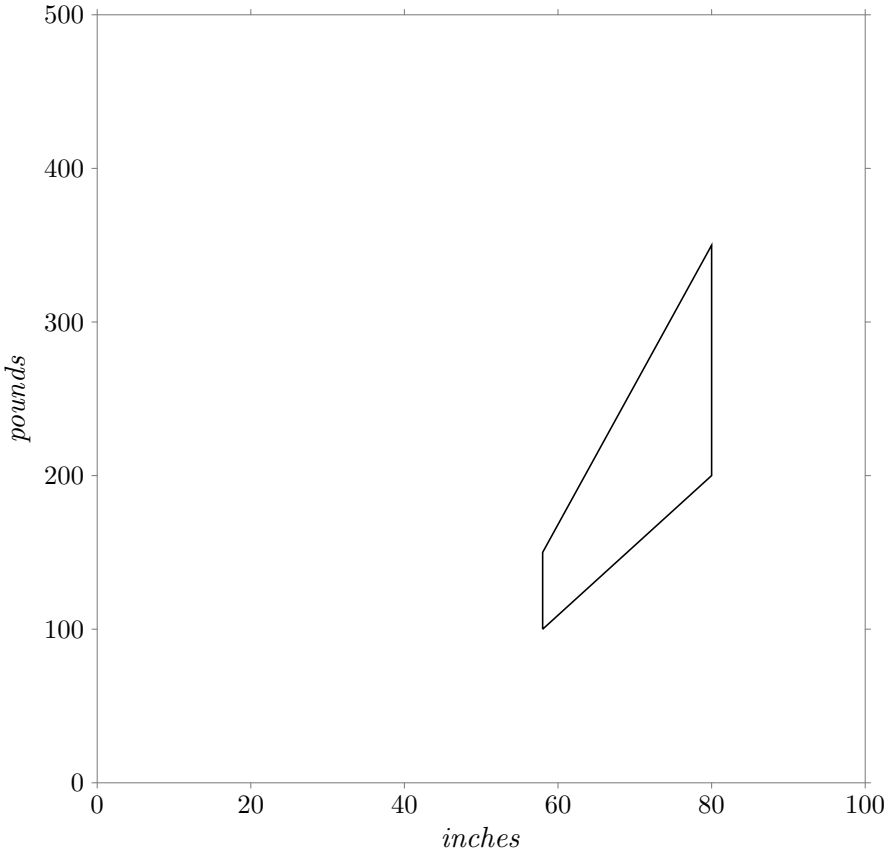

Figure 5.1: Human Males: Likely Heights and Weights

;

There is no magic in two characteristics. Various other bodily measurements can be added. We can add various characteristics to distinguish in ways important or trivial or misleading. For example, craniums of different shapes, of eyes of different colors or acuity, of hemoglobin selected for life at sea-level or on the slopes of the Andes Mountains, and so forth, can be studied in a disciplined and reliable way. Conceptually, it is trivial to expand the concepts from 2 characteristics to 5 or 100 or 1,000,000 or an indefinitely larger number.

At this point, the use of some sort of states of being might seem to lead to no better than a more elaborate, and far more plausible, form of quantitative reduction of a complex entity to its parts and aspects. But this is where we begin to see possibilities for the machinery above for dealing with qualitative or abstract being as well as quantitative or concrete being. We might not be able to create explicit graphs, though we might well be able to produce graphs of quantitative variables for some samples of qualitative possibilities. In fact, economists and political theorists do something such as this when they look at one economic model for a community when members trust each other, one for that same community when members distrust each other, and some for that same community with different levels of trust and distrust. In the long-run, we would aim to encapsulate such plausible solutions into an understanding of complex entities as being yet unified and coherent and complete.

I don't know how to carry out these projects, or the one grand project of re-understanding Creator and Creation. I do know that such projects are best carried out one step at a time though we have to keep our eyes on some sort of long-range goal. I also know that my fellow-Christians speak in the language of Gibber even when they deal with clear truths about, say, human moral nature. They've thrown away all chances of speaking of human beings as true entities; they prefer by habit and by fear of modern knowledge or indifference to modern knowledge to speak of an implausible concatenation of soul and body, of a mortal and defectively moral being trying to recover from the fall of Adam and Eve. Too many conflicts of particular knowledge—though not of moral truths—have occurred because some parts and some aspects of human being are increasingly subject to scientific investigation which produces exact and, sometimes, reproducible results. Our understandings of human being and even our ways of stating the most absolute of moral truths have to correspond to what is truly known about man in his concrete parts and aspects. Many who would honor tradition talk as if man's moral nature fell from some state of perfection while the rest of our parts were evolving and developing from an ancestor common to human beings and chimpanzees. Meanwhile, the soul or mind has become a wretched little thing retreating to successively smaller regions of human being which haven't yet been explored by modern brain-scientists.

Sometimes, Christians have not just been derelict of their duties to understand God's Creation, to encapsulate God's manifested thoughts as our own. Sometimes, Christians have persecuted their own members who have

tried to reach the better and more exact understandings of parts and aspects of this world which became possible as sciences such as physics and the technological arts developed into important parts of human life. The most spectacular battle against a man trying to learn the thoughts of God in His freely chosen role as Creator was waged by the Roman Catholic hierarchy against Galileo, himself a devout Catholic and, arguably, a better Catholic theologian than the Churchmen who persecuted him, even the Jesuit Robert Bellarmine who was a brilliant political philosopher if defective theologian and Bible scholar.

Despite the mistakes of those theologians arrayed against Galileo, it is true that theologians and metaphysicians have some attitudes better attuned than scientists to the study of human being and the universe and all of Creation as complex entities. Strangely enough, there are no scientists to my knowledge, not even among those philosophically inclined, who have even suggested using these powerful techniques from modern mathematics to study complex entities such as human beings; of course, most scientists—even those who are practicing Christians or Jews—seem to be believers in modern ideologies of individualism and don't seem to assign any reality to communal forms of human being nor even more than a formal, metaphysical reality to the universe as such. To be sure, as is usually the case, there are exceptions: see an early essay on my website, *A Universe is More than it Contains* found at `http://loydfueston.com/?p=163`, for a pointer to a discussion by the cosmologist and astrophysicist, P J E Peebles, of a probable (small) violation of the law of the conservation of energy on the level of the universe; this violation indicates rather strongly that the universe is something different from **just** a collection of its parts. The most relevant part of that essay is:

> We see that the faster decrease of [the radiation density of a relativistic universe modeled as a gas] compared to the mass density of a nonrelativistic gas is the result of the pressure work done by the expanding radiation. However, since the volume of the universe varies as [the third power of the expansion factor of the universe], the net radiation energy in a closed [and expanding] universe decreases as [the inverse of the fourth power of the expansion factor of the universe—due to the Stefan-Boltzmann law] as the universe expands. Where does the lost energy go? Since there is no pressure gradient in the homogeneously dis-

> tributed radiation, the pressure does not act to accelerate the expansion of the universe. (The active gravitational mass due to the pressure has the opposite effect, slowing the rate of expansion...) The resolution of this apparent paradox is that while energy conservation is a good local concept...and can be defined more generally in the special case of an isolated system in asymptotically flat space, there is not a general global energy conservation law in general relativity theory. [*Principles of Physical Cosmology* [24], P J E. Peebles, page 139.]

A disclaimer: we don't yet know the exact geometric properties of the universe, or rather—the region of the universe which is visible to us. It's not even certain we have a complete inventory of types of matter and forces in this universe. It might be that we do live in a universe that conserves energy at the 'global' level, but there's no 'law' mandating such and a good chance that we live in a universe where certain types of energy can go away at the level of the universe.

On the whole, scientists do far better at their particular tasks than modern theologians and philosophers do at theirs. The reductive skills of scientists can dangerously distort their view of creatures at times, but those skills also allow scientists to be quite good in defining what the true parts and aspects of a man are and pretty good at studying many of those parts and aspects. Yet, from a global viewpoint, that of a man as an entity which is kind-of unified and not very complete and defectively coherent, science fails as the field of thermodynamics may fail at understanding the universe as a whole. Julius Caesar wasn't just a freestanding man, he was a Julii and a Roman—he was, in a strong sense, Gaius Julius Caesar and also the Caesar branch of the Julii and Rome itself and even *Homo Sapiens Sapiens* and so on up to Creation and the Son of God through Whom and from Whom Creation was made. Most importantly to a Christian, he was all of that in response to Creation and to its Creator.

Evolutionary biologists wage major battles every few years as one side says, "Group evolution is for real," and the other side says, "No, it isn't." One side claims victory and a few years later, that victory is lost or is at least uncertain. Some of this back-and-forth and other sorts of confusion might be settled if scientists, including evolutionary biologists, tried to adopt concepts and techniques from mathematics. They might even learn there are reasons for some people believing in immaterial beings—those

ghosts and angels and demons play the role assigned by me to complex mathematics which 'explains' what are often mysterious effects of global beings, including the 'life-force' of individual living creatures.

As much as I support the efforts of evolutionary biologists and geneticists, as much as I accept the results of their research, I claim they aren't thinking very clearly when they try to make sense of it all, where 'all' refers to the entirety of human being. Parts of what might be called human reality are well-described by that research. But, parts of human reality—such as the bonds of human beings with their various communities—are not rendered void because they don't fit in with the theories of those scientists. A theory has to conform with reality—a truth well exemplified by most research in evolutionary biology and genetics but not by denials of the reality of human communities, or wolf or even crocodile communities. If a theory can't match up to the fullness of reality, then work on the theory or admit parts of reality lie outside of that theory; don't deny reality because it doesn't meet the standards of your theory. This is the sort of thinking that led Bellarmine and others to persecute Galileo.

Whether the global 'shape' of man is that of a fully natural creature, that of a creature of body and soul, or—as I believe—a fully natural creature capable of responding to the Creator by learning how to share His thoughts, it's possible to use modern, often abstract and qualitative, mathematics to gain a greater, a global, understanding of man as an entity who has various parts and aspects, some subject to strictly quantitative study and some not so subject and maybe not at all subject to quantitative study.

We reduce man to his parts and aspects and we abstract upward to communal human being. Physicists do something similar: reduce to fundamental particles and relationships; then jump upward to spacetime and to a particular universe and to... Move back and forth between bricks and the building as a whole.

The above paragraphs are preliminary, and quite inadequate, statements of a viewpoint which can't be fully developed or fully justified in even the entirety of a large book. All we can do is wander in a general direction—there isn't even a path until we travel it, one step at a time. And we must remember the nature of this world in order to understand the contingent and personal nature of this path. Since I'm traveling it after centuries of stagnation of Christian thought about Creation, I get to choose the path within certain limits known only to God and perhaps suspected by me.

What will be the result of traveling such a path?

First of all, we can heal, can pull together our fragmented individual selves—if only in a speculative way. We do this by way of understanding and accepting that these fragments can form a more global self—unified and coherent and complete to the extent possible in this mortal realm.

Secondly, we can begin to understand human communities as being unified, coherent, and complete collections of individual human beings and, maybe, other creatures and other stuff.

Thirdly, we can begin to behave as unified creatures, as Christians in all that we do and at all times of the day and week and year.

Is there any form of being, or any aspects of concrete being, which can be truly non-quantitative? Are the categories of qualitative and quantitative truly mutually exclusive or even largely so? Do they truly overlap, maybe to the extent of one being a subset of the other? If the fears of some mathematicians are right, it's possible that the quantitative arises from a layer of highly abstract, qualitative being, but I don't really know the answers to the above questions and consider them difficult enough to avoid proposing firm answers, though I believe that there are forms of abstract being which are non-quantitative and also believe those forms of abstract being flow into human being as surely as do the forms of concrete being which are the result of so-called collapses of quantum wavefunctions.

5.2 Qualitative States

The modern world is hard to figure out. Even as critics in the West unleashed artillery barrages against the materialism of Marxists of all sorts, the Anglo-American world was slipping into various sorts of materialistic thought, though often running alongside strange forms of 'holism', often based upon what were probably misunderstandings of Hindu and Buddhist thought. At the same time, a great mathematician living under the ideologically grounded terror of Stalin and his bureaucratic successors, could write:

> The proponents of mechanistic materialism assumed that such a formulation [of systems describable in terms of relatively simple differential equations, such as gravitational fall] is an exact and direct expression of the deterministic character of the actual phenomena, of the physical principles of causation. Ac-

> cording to Laplace, the state of the world at a given instant is defined by an infinite number of parameters, subject to an infinite number of differential equations. If some "universal mind" could write down all these equations and integrate them, it could then predict with complete exactness, according to Laplace, the entire evolution of the world in the infinite future.
>
> But in fact this quantitative mathematical infinity is extremely coarse in comparison with the qualitatively inexhaustible character of the real world. Neither the introduction of an infinite number of parameters nor the description of the state of continuous media by functions of a point in space is adequate to represent the infinite complexity of actual events. [A N Kolmogorov, *The Theory of Probability*, published in *Mathematics: Its Content, Methods, and Meaning* [21].]

A N Kolmogorov (1903-1987) was a true giant among 20th century mathematicians, having had a long and distinguished career as teacher as well as creative mathematician as well as writer of important textbooks which remain in publication and use. And he was a man who spent his entire career under Lenin and Stalin and their successors, dying while Soviet ideology was also dying but still thrashing about. If he could see, and write, that quantitative aspects and parts of reality are embedded in a greater qualitative whole, then I think it odd that the West struggles, and often doesn't bother to struggle, against a simpleminded materialist (quantitative) understanding of what lies around us. This might be the reason for the lack of interest in Western thinkers in exploring, in a rational way, the greater possibilities raised in modern science and mathematics, such as the clear possibilities that there are greater wholes (the universe itself or a human being) with properties that cannot be derived by any straightforward 'summing' of the parts.

Is anything in this concrete world really qualitative in a strong sense? It's certainly true that not all things (such as beauty) are reducible to a simple and undeniable numerical scale, but—given an appropriate context—some sort of good ordering is possible for most, maybe all, sorts of beauty. The differences between Igbo and Norwegian standards of beauty for women, the differences between those who place the highest priority upon symmetry and those who think some non- or even anti-symmetry adds to beauty, can be handled with multi-dimensional sets of sets (of sets of...).

We can provide a good ordering within the lowest-level of sets and then deny that ordering is necessary or even possible above the lowest-level; or something like that. Similar statements can be made about paintings and statues and buildings, about ways of thought, about moral goodness or moral understandings.

Anything which can be perceived, can be responded to, can be thought about, can be somewhat quantified, if only ambiguously and, often, if only after the fact. By "after the fact," I refer to at least the way in which the acts of creatures with some greater or lesser amount of moral freedom can be quantified, even in ways which falsely imply the morally free creature wears the chains of a deterministic world. Human beings are organisms of the sort which are constrained in many ways, but not fully enchained by any means; they do move in environments which direct their paths in various ways and prevent them from moving in certain directions, at least not without great ingenuity of the sort realized only in morally well-ordered civilizations. Alas, any human community, even the great cities—'cosmopolises'—at the heart of great civilizations, will itself constrain freedom in some ways even as it opens up more free movement in other ways.

In the most abstract regions explored by modern mathematicians, not all types of spaces are quantitative. For example, we can't always calculate the distance between two objects, x and y, in a set, U, found in that space. Not all are even quantitative in the sense that we can place those two objects, or the other objects in that set, in a good order. I'm speaking in a somewhat sloppy way to make the point that mathematics isn't so restrictive in the sorts of spaces, sets, objects, etc which can be considered, not so restrictive as many sciences which draw upon parts of mathematics which are strictly quantitative, not so restrictive as many who would reduce all to what can be calculated in a conventional sense.

Let me take an example of philosophical reasoning to be found inside of mathematics. Most modern men, though not necessarily those of some other times, would agree that all points are the same, such as the points on continuous graphs. If so, goes the claim, then there is only one point and other points are representations of that point. I originally read of the idea in the article, *A Mad Day's Work: From Grothendieck to Connes and Kontsevich the Evolution of Concepts of Space and Symmetry* [4]. Cartier tells us:

> If all points are *intrinsically* indistinguishable from one an-

> other, they can differ only in *position*. In other words, there exists an archetypal point, of which the other points are *representations*.

This seems to be similar to the claim made by the physicist *John Wheeler* (see `https://en.wikipedia.org/wiki/John_Archibald_Wheeler`) in a phone-call to the physicist *Richard Feynman* (see `https://en.wikipedia.org/wiki/Richard_Feynman`). This is discussed in the short article, *One-electron universe* at `https://en.wikipedia.org/wiki/One-electron_universe`. John Wheeler wasn't known for being shy about proposing radical ideas in physics or in the philosophical understanding of physics, such as the 'reality' behind electrons. (Reality in this last sentence has to be taken in a sense at least quasi-Platonic, whatever that might really mean.)

At this point, you might be realizing that I've gone off on a real tangent, but there's a purpose for this digression. So, please bear with me for just a short while. I'll return to the subject of this section: *Qualitative States*.

The ancient (1913) but solid edition of Webster's dictionary [29] gives us a number of definitions for 'point'. Two that seem relevant to this discussion are:

> **point** 5. An indefinitely small space; a mere spot indicated or supposed. Specifically: (Geom.) That which has neither parts nor magnitude; that which has position, but has neither length, breadth, nor thickness, – sometimes conceived of as the limit of a line; that by the motion of which a line is conceived to be produced. [1913 Webster]

> **point** 6. An indivisible portion of time; a moment; an instant; hence, the verge. [1913 Webster]

How useless for understanding what a point really is. These are axioms, statements of assumed entities. We build from points but we don't penetrate and understand them. Most relevantly, we can't put a point in a quantitative relationship to much of anything. Not even to a line. It is "sometimes conceived of as the limit of a line," and that can lead to some good, disciplined ways of math-speak which verge on understanding but it is a **qualitative** understanding.

A point is something which is not really defined, not in modern mathematics. It can't be since it can be an entity in an abstract set or in,

for example, a topological space so abstract as to have no part with such concepts as distance or even order. Even when there is some meaning assigned to, say, $a < b$ with $a, b \in A$, where A is some topological space, that '$<$' might mean something greatly different from what you learned in high school or undergraduate college mathematics. A point doesn't necessarily have even position, it's simply the most basic, even atomic, element in a set or a mathematical space. Before going on, I'll warn the reader that "most basic" doesn't imply any sort of absoluteness but rather speaks of basic within the context of a particular set or space or other thingy. A point itself can be a set or an entire space with its own internal structure or lack of structure.

The real questions aren't the likes of:

- Do qualitative states exist?
- Is anything really qualitative?
- Does any concrete entity have qualitative parts or aspects?

The real questions are something like:

- Is some sort of qualitative being, of a very abstract sort, the basic or raw stuff of created being?
- Are things of a quantitative sort created from qualitative being?
- Does qualitative being continue to exist in this world of concrete things?

I'll tentatively answer, "Yes," to each of the above three questions, though the tentative nature of my answer has to do more with my way of advocating it and I strongly believe in that, "Yes."

It would seem that some mathematicians prominent in the development of modern, very abstract mathematics had fears or hopes that such is the case and it was learning of these concerns that led to my perception of the possibilities of using reasoning processes discovered by modern mathematicians to give modern Christian thinkers a chance to regain a properly rich and properly complex understanding of being, including that of this

concrete and highly particular universe, in the context of a Christian understanding of what it all means. Yet, the prior sentence will imply an erroneous view to those who haven't understood the spirit of my writing.

The human being, heart and hands and mind, should live in active response to that which surrounds him and also that which is him or in him. Our responses shape us, individuals and communities, and those shaped human beings develop, sometimes implicitly and sometimes explicitly, an understanding of what exists, what is contingent and perhaps what is transcendental—above this mortal realm of flux in some sense of 'above'.

Knowledge isn't a collection of data, but rather a history of active responses to being, responses which are themselves a form of created being. Knowledge is an encapsulation of the being which we can perceive or conceive, an encapsulation formed by active responses to that being.

Part II

The Qualitative Beneath and Beside and Over the Quantitative

6 What is an Entity?

What are these complex human beings, individual and communal, that I'm trying to analyze? Are they really similar in some fundamental ways to other sorts of complex entities in this concrete, thing-like universe?

Does a star have properties which subsist through its entire existence? Am I entering some sort of vicious circle where a star is a thing which has subsistence properties corresponding to those of a star while the properties of a star are those which subsist in some specific things?

Does a star have a well-defined boundary? If we were inside the star, could we define a limit-sphere (a two-dimensional equivalent of a limit-point) and say that the star ends there? Up to that point is 'starness' of this particular star and beyond is 'non-starness' and other than this star. What about the gases which dance inside, near the boundary, and outside that boundary? Do they cease to become part of the star when they dance out into space and then once again become part of the star when they fall back? What about the electromatic pulses it sends out and the somewhat more stable magnetic fields which extend beyond what seems visually to be the star? Is the curvature of spacetime (gravity field) produced by the star part of it?

How about a bacteria? We can consider one interesting question. Bacteria live in a specialized biosphere which occupies the same space as other creatures such as human beings, but that specialized biosphere contains a variety of bacterial genes including ones which confer resistance to various natural and man-made antibiotics. Individual bacterium contribute these various genes and take them in when needed. (The process by which a bacterium 'finds' a gene giving some or full resistance to penicillin is mysterious to me, largely because I don't have much interest in the details nor much background knowledge to learn about those details without a bit of time. As usual, there is a Wikipedia article,

https://en.wikipedia.org/wiki/Antibiotics, which discusses antibiotics in general including a little about the problems with resistance and the ways in which bacteria exchange genes so useful to them.) If 10% of a bacterium's genes are the sort which are shared, exchanged in some sort of bacterial community, could we say that bacterium is 90% an individual and 10% 'absorbed' into a community?

The situation is far more complex with human beings. I'll merely point to the events of human individual and communal lives as recorded in our personal memories, in immediate forms of journalism, in history, in philosophy, and in theology. A serious effort to learn a significant bit of neurobiology, evolutionary biology, modern medicine, and so on will lead us to think these sciences can help us to understand human being in its two forms, individual and communal, but the complexities of a beer-brewing, poetry-writing species in contact of some sort with the Creator go far beyond the complexities of any other known species—most certainly beyond the complexities of bacteria.

The motivation for this book? I find little reason to believe we have coherent ways of discussing human being, individual and communal. In fact, we have no good ways to talk, in modern scientific and exact terms, about human communities as being real and not just nominal entities which are really just voluntary gatherings of freestanding individuals. As some have seen, the very foundations of a Christian understanding of this world and of all of created being, Creation, were shaken and severely damaged by two very dangerous, anti-Christian ideas:

Nominalism A philosophical view which denies the existence of universals and abstract objects, but affirms the existence of general or abstract terms and predicates. [See https://en.wikipedia.org/wiki/Nominalism]

Univocity of Being The idea that words describing the properties of God mean the same thing as when they apply to people or things, even if God is vastly different in kind. [See https://en.wikipedia.org/wiki/Univocity_of_being.]

For the background of the thinkers, start by checking on John Duns Scotus at https://en.wikipedia.org/wiki/Duns_Scotus and William of Ockham at https://en.wikipedia.org/wiki/William_of_Ockham). It's

interesting but not that surprising that both men were devout Franciscans but didn't seem to even suspect that nominalism would cast in doubt such Christian truths as the reality of human communities, including the Body of Christ, and univocity of being would cast in doubt God's transcendence and His role as a true, freely-acting Creator. In a word, either concept and certainly both together would seem to be irreconcilable with Christian views of created being as sacramental and as created by, sustained by, and blessed by the Creator. And I would suggest that without these philosophical positions, both coming from a school of thought in a religious order of the Roman Catholic church, modern forms of reductionism and materialism wouldn't have invaded western thought so thoroughly that, as I noted in a precious chapter: a mathematician writing from inside of the Soviet Union with its highly regulated, Marxist thought could be more appreciative of the reality of the immaterial than are most secular or Christian thinkers of the modern West—see page 63.

Over the years, I've been reading and still read books in sociobiology and neurobiology, anthropological prehistory and ancient history and more recent history, political philosophy and moral philosophy and philosophy of history and philosophy of science, physics and mathematics, and much literature of the sort which contains serious ideas and profound feelings of the truly human sort. I've found no reason to believe that scientists or historians or philosophers or theologians or even great novelists could handle the complexities of human communities as they have developed by the early 21st century after Christ.

We need new concepts, new words, and new ways of phrasing ideas to be able to see the patterns in the complexities of modern human communities and any other extremely complex created entities which we might find and have reason to study. We need those new concepts to discuss our human being, individual and communal, with others. We need them to more reasonably identify our problems and opportunities and to explore ways of dealing with those problems and opportunities so often the same or at least heavily overlapping.

Let's continue creating some of what's needed.

7 Describing Being in Our Concrete and Abstract World

7.1 Seeing the Invisible

In Part I, *Laying Some Foundations*, I covered some motivational issues—not only what but why—and also dealt with some broad-brush issues such as the nature of abstract spaces as used in modern science and mathematics. Now in this Part II—*The Qualitative Beneath and Beside and Over the Quantitative*, I'm dealing with a few issues of particular concepts which can help me to move improve the Christian worldview I started constructing 20 years ago or so. To remind the reader, that worldview is intended to be a fresh and properly rich and properly complex understanding of Creation. I'll continue touching upon specific, concrete issues which arise when we Christians respond to Creation in the context of the modern world.

First, let's step back and examine the general framework in which particular forms of being, concrete and abstract, can be displayed in a tree (turned upside-down) of descent—as an evolutionary theorist might say, though 'bush' rather than 'tree' is more popular in recent years, for good reasons, though we might want to add this is a bush in which branches sometimes merge—such as the matings of anatomically modern human beings with various archaic human beings, including Neanderthals and Denisovans and perhaps some truly primitive human beings. Scientists claim that the DNA of those of European descent contains about 2% Neanderthal genes. Eastern Asians and some other peoples have a little more, though that includes other archaic human beings similar to Neanderthal human beings.

How to make sense of all of this?

I'll refer to the following graph I've used before:

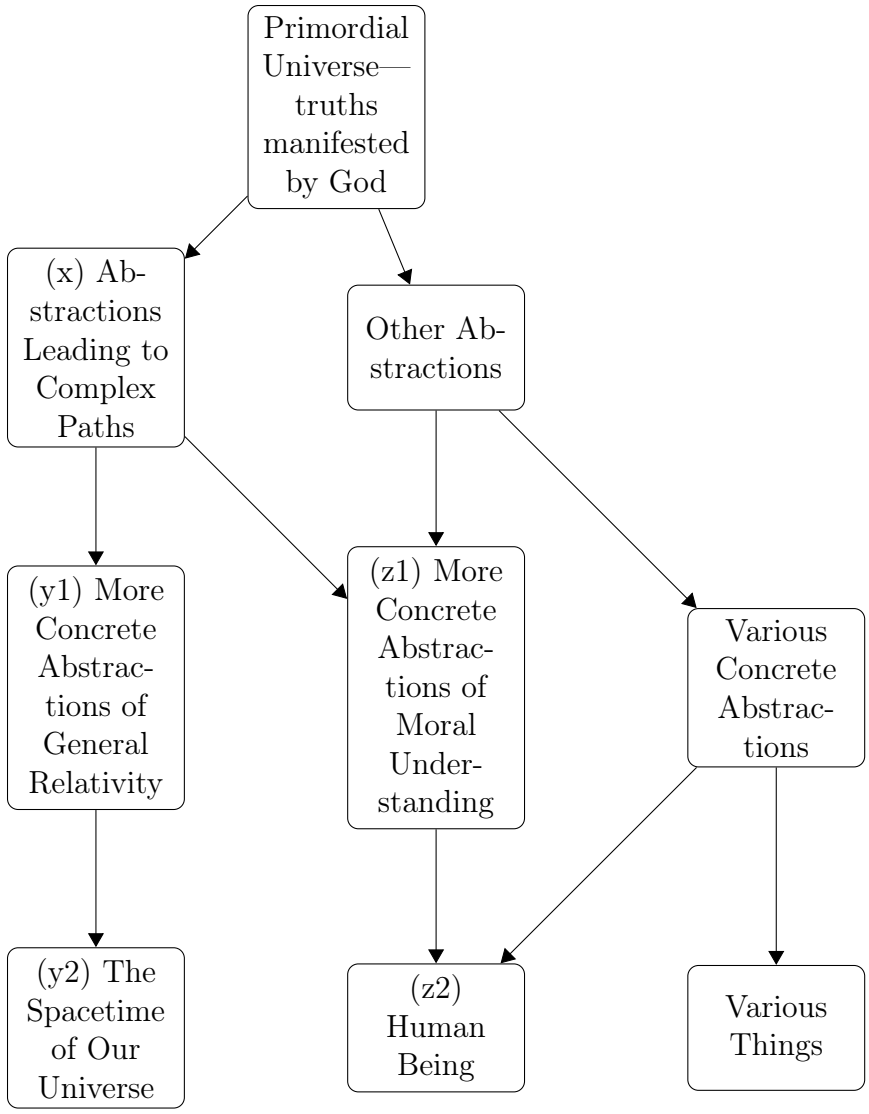

Figure 7.1: Simple Relationships of Abstract and Concrete Levels of Being

We can start at *Node y2*, which represents our current understanding of *The Spacetime of Our Universe*. This node is found on the bottom row. From that node, we work our way up through higher levels of abstraction

until we reach a level which shows some promise for helping us to understand, for example, human being. So, by abstracting somewhat, we can reach *Node x* which is *Abstractions Leading to Complex Paths* and then travel down to *Node z2* which is *Human Being* including our understanding of our moral pathways through this world. This understanding is also supplemented by other abstractions as shown by the arrow from the unlabeled node titled *Various Concrete Abstractions*. By *concrete abstractions*, I intend to convey the idea of a level of abstract being which is close to that of our concrete world.

The above graph is almost a cartoon, but one which provides an aid for our all-too human minds. That graph, or cartoon, is a picture of my claim that the thing-like being of this universe is **not** just the matter subject to analyses of particle physics at the fundamental level or the analyses of evolutionary theorists and geneticists at a much higher level of organization. It is also all that strange, immaterial stuff subject to misinterpretation, or perhaps it be better to say misexplanation, of so many over the centuries. But at least they got it right that the world is more than one of billiards balls colliding and bouncing in this direction and that, more even than the more complex world of nonlinear physics—chaos and complexity and all that.

With that in mind, I'll be throwing out some mathematical concepts with some speculations about how they can be used in the task of understanding human being in all its complexity and richness, as well as the being of boa constrictors and plutoids (poor demoted god of the underworld) and black-holes and transfinite numbers [1].

In a sense, the above explanation is somewhat distorted as the primary form of being is relationships—we should strive to understand relationships before we can understand the concrete entities or the simpler forms of thing-like being in this universe. We can learn how to do this from quantum mechanics and from evolutionary biology—the environment provides for relationships for which organisms are selected. We can learn how to do this by studying modern mathematics in all its abstract strangeness. I'll finish preparing for a little bit of mathematics by speaking of the human mind, always and necessarily shaped so that it sees some things and sees right

[1] As I've said often in my various writings, even the most abstract of entities is a created being from our standpoint because we wouldn't know about them if they were not in the truths manifested by God as the raw stuff of Creation.

through or right over other things, assumes some thoughts which can at least be rational and can't even parse statements of other thoughts which might be more rational and maybe more true to created being.

7.2 Intuitively Approaching a More Complete Understanding of Created Being

Let's take a trait of created beings, of complex entities with at least a primordial sort of moral nature—gratitude which is itself complex, penetrating into the parts and aspects of those creatures of moral nature. Human being can be usefully explored and discussed in terms of mind and heart and hands—see *A More Exact Understanding of Human Being* at `http://loydfueston.com/downloads/human-being.pdf`.

Imagine we have in front of us a grateful woman. Imagine further that we have a lamp that can project a image of her mind and another lamp that can project a image of her heart and another that can can project a image of her hands (in the general sense of her physically active 'parts'). Each of these three images can be examined, explored, and described in disciplined ways, not necessarily all in the same way.

There is an interesting question here: Can the images be mapped back to form an entity which is, who is, that grateful woman? Not necessarily. First, let me return to topology to prepare for an admittedly uncertain answer to this question.

> The study of topologized sets (or any other abstract system) involves two broad and interrelated questions.
>
> The first of these concerns the investigation and classification of the various concrete realizations, or models, which we may encounter. This entails the recognition of equivalent models, as is done for isomorphic groups or congruent geometric figures, for example. In turn, this equivalence of models is usually defined in terms of a one-to-one reversible transformation of one model onto another. This equivalence transformation is so chosen as to leave invariant the fundamental properties of the

> models. As examples, we have the rigid motions in geometry, the isomorphisms in group theory, etc.
>
> ...
>
> The second broad question is studying an abstract system such as our topologized sets involves consideration of transformations more general than the one-to-one transformation. The requirement that the transformation is one-to-one and reversible is dropped and we retain only the requirement that the basic structure is to be preserved. The homomorphisms in group theory illustrate this situation. In topology, the corresponding transformations are those that preserve limit points. Such a transformation is said to be *continuous* and is a true generalization of the continuous functions used in analysis.
>
> [See pages 2-3 of *Topology* [18] by Hocking and Young.]

Ultimately, for abstract sets in which category I put qualitative sets, the possibility of a topology and, thus, of the application of topological reasoning, concerns mappings which preserve limits, so that limits in the region of the manifold which models that grateful woman remain limits in the 'flatter' and easier to analyze region of Euclidean space or some equivalent sort of abstract or qualitative space.

So, I return to the question: Can the images be mapped back to form an entity which is, who is, that grateful woman? I suggest, "Yes," though we can't expect any exactitude which is to be found in something such as a mapping of the earth's surface to Euclidean spaces and a reverse mapping from those Euclidean spaces back to the earth's surface as we know it on a spinning globe. There are even two closely related and dangerous examples of this sort of conceptual or qualitative mapping: Christian trinitarian theology and the anthropological-theological understanding of the Body of Christ, flesh-and-blood and divine but not so much mystical.

I'm going to introduce a complication here, one I'll deal with in an upcoming example from mathematics: when we deal with the 'unbundling' of 'regions' of complex entities, we might be dealing with components or aspects which are not strictly regional in a geometric sense.

Let me take up a matter of pure theology, the Christian understanding of God in His transcendence:

> God is a divine manifold, in a manner of speaking. The light

> of revelation as best understood by human wisdom of the age shines upon God, also in a manner of speaking, and can project an image of Father. Let the wise man move a bit to catch the divine light of revelation from a different angle and the image is that of Son. Move a bit more and the image of the Holy Spirit appears.

There are no separate regions on the divine manifold. And so it is that we have some trouble seeing how it is that the image of God which is the Father represents the entirety of God, as do the images of Son and Holy Spirit. Moreover, though it seems almost blasphemous to use such words: we have a problem in not being able to probe and explore the divine Being, but this points to a general difficulty due largely to my gross simplification of what is involved in the sort of understanding which I propose. I could correct my wrongful ways of speaking by noting that I'm speaking of matters where we understand but can only sometimes probe or explore. In some cases, such as our efforts to understand God, we can only try to understand what is given to us.

Let me provide a simple mathematical example of a case where a set, the real numbers, $\Re$, is a unified entity and yet it can be broken down into three subsets which are also unified entities but are mixed in that greater set in such as way as to force strong qualifications, not to be provided here, on what it means to separate them.

Define three sets which, in total, are the real number line.

- A $= \{x \mid x$ is an integer$\}$.
- B $= \{x \mid x = y/z$, where y, z belong to A but x doesn't belong to A$\}$.
- C $= \{x \mid x$ belongs to $\Re$ but x doesn't belong to A or B$\}$.

Though these numbers are part of an infinitely rich structure which is a unity in a meaningful sense, the entirety of the real number line, we can produce sets which are three more or less natural components of that line, of that set of all real numbers. 'Natural' in this context means: intuitively appealing the human mind. The reader who's accustomed to reading mathematical or scientific works will probably quickly translate the above list into words and concepts. For other readers: The three sets are integers—A, rational numbers excluding integers—B, and real numbers excluding rational numbers and integers—C.

That's all. There is nothing magical about what I'm advocating. Creation includes complex entities which can be, from a true but limited viewpoint, reduced to more basic entities though the complex entities have themselves true being. There are many who claim, if only by their political and economic and moral actions, that human communities are but gatherings of freestanding individuals who have more of a contractual belonging than an ontological belonging to their communities. Rare indeed is the human being willing to say that those 'freestanding individuals' are nothing but collections of cells cooperating for the sake of their survival and controlled by DNA which uses those cells and the human organisms and the human communities to produce more or less faithful copies of their own biochemical selves.

We don't need magical ways of showing that human communities are... communities and not just voluntary gatherings of freestanding individuals just as we don't need magical ways of showing that individual human beings are such and not just ad-hoc collections of more fundamental biochemical entities. And we can see this is analogical to rigorously explorable sets in mathematics which are truly entities with their own properties though they are also made up of components of subsets which each have their own defining properties.

Human individuals present themselves to our perceptions and conceptions as human beings, complex entities which are a particular type of created being. Human communities present themselves to our perceptions and conceptions as human beings, complex entities which are a particular type of created being.

I never promised what so many modern thinkers falsely promised. That is, I never promised there is something out there called 'knowledge' which is in principle something to be grasped at, something to be rendered submissive to a strange entity called a 'human mind', a strange entity which floats free of being—created and divine, a strange entity which can render judgment upon that being as if standing outside of Creation and standing above God Himself. What I would call true knowledge is the shaping of the brain to encapsulate some lesser or greater part of the being which lies around us. And our brains are predisposed and nearly always properly shaped to take human being, individual and communal, as real.

As a Christian, I am particularly concerned with the ultimate human community: the Body of Christ. This Body is evolving and developing from the bottom-up, for the most part; yet, it is evolving and developing under

the guiding constraints and pressures of a highly particular universe which is itself both a specific entity and container for other entities—all inside a peculiar Creation, one arising from a manifestation of truths through the Word, the Son of God.

A mouthful, an abstract mouthful at that. It's hard to put even partly new ideas in words easy to digest, but let's at least try to make a somewhat more concrete description of the Body of Christ in terms similar to those used above in my description of the Holy Trinity:

> The Body of Christ is a flesh-and-blood manifold in union with God through the man-God, the incarnate Son of God. The light of human knowledge and wisdom, based upon the best knowledge of the age, shines upon the Body of Christ, the ultimate community of men, and projects images of a complex and kaleidoscopic variety—individuals and communities of human beings, political and religious and economic and athletic and academic and labor-centered and so on.

We can have our cake and eat it too. That is, we can be unified as communities and remain individuals; we can be unified as individuals while recognizing the fragmentation which is our embodied selves. This is not merely a matter of developing a way of speaking, though it begins with that. The mathematics which allows the mapping of a community into its sub-communities and its individuals, the reverse mapping which puts a those parts into the whole, reflect true forms of created being and true relationships between those forms of created being. This mapping is what I've described as "shining a light" which light is knowledge understood as an encapsulation of being in human minds, individual and communal. That light is itself formed by that process of encapsulating being and then can shine forth to begin another iteration of active understanding and feeling and doing followed by re-encapsulation in response as some of that activity, some of those acts-of-being, match up well with created being as we can reach it and some don't match up so well.

The relationships between individuals, between individuals and sub-communities, between individuals and the entire community, between sub-communities and the entire community are real and shape all of these entities.

Part III

The Totality of Human Being: Individual and Communal

8 What Are Our Problems and Opportunities?

8.1 The Poverty of Modern Thought Relative to the Known Richness of Human Being

We started and continue with a major problem: we human beings all wish to be unified creatures but we seem to be bundles of incompatible sorts of things, traits and characteristics and tendencies and structural elements and whatever. The language of description for this situation can vary and even the conceptual underpinnings can change depending upon the domain of the discussion. For example, we are historical creatures and moral creatures and biological creatures and so on. Each of those major parts of human being has many subparts. We also have a similar problem in understanding our communities. We have no ways of speaking or thinking about our individual selves and our communities, our communal selves, as both existing without the individual tearing itself from the community or the community absorbing the individuals so that they are individuals only in the sense that individual cells make up the human individual body.

Most efforts to look at human beings or other living creatures start with quite legitimate efforts to understand some part or aspect of a being, human or other. This isn't an attempt to categorize before understanding, nor is it—in general—an attempt to reduce human being to being just 'anything'. Provisional categorization is important in all forms of human thought and it is remarkable how capable the human brain is at realizing, even in an infant just acquiring a vocabulary, that a dachshund and a Great Dane are both 'dog'. According to some linguistic analyses, the human brain is inclined to start classification at the biological level of genus rather than species so

that a wolf and a fox would also likely be 'dog' to that young genius [1].

Historically, it has been those not specializing in narrow fields, such as theologians and philosophers and poets and novelists, who have made greater sense of it all. Call those generalists top-down thinkers and the specialists, such as biologists and physicists, bottom-up thinkers. (Some, such as historians and probably some specialists such as philosophical physicists, seem to have a switch to put themselves into one mode or the other, top-down or bottom-up. Einstein and Newton, probably Bohr, started some of their major projects with philosophical contemplations.)

In his book, *The Unintended Reformation: How a Religious Revolution Secularized Society* [15], Professor Brad S Gregory tells us that we need to have an overall Christian understanding of the world if we are to once again have a Christian community much like Western Civilization. This is a job for an intellectual elite, one which will refuse to travel the ruts of the current intellectuals of the West, one which will rebel against the established institutions, reforming or replacing them. As Carroll Quigley noted in *The Evolution of Civilizations: An Introduction to Historical Analysis* [25], institutions and their inmates become part of the problem, part of the obstacles to change, during troubled times. To be sure, he did claim that sometimes they are reformed but often they are so self-serving and so stuck in their ways that they must be passed by as they die or more actively destroyed. There are currently a lot of resources tied up in various human institutions which are self-serving in that way. Those institutions and their individual members are not about to take the sorts of risks which their founders had taken.

Christianity has both practices and beliefs which are quite sophisticated and quite complex. It can place heavy demands upon mind and heart and hands, the entirety of human being—individual and communal. It's the demands upon the mind which seem most at issue in the case of Christianity, despite the common falsehood that we have only to do good, to let others see us doing good, and then many will be flocking to the baptismal font, many others who drifted away will flow back into one or another Christian Church.

[1] Brain-scientists often note the extremely high ability of young children to learn the language which surrounds them even if they show no later ability, whatsoever, to learn a foreign language. This is particularly notable since those young children have some not yet understood human-language structure to their brain—but still have to learn a first language even if such concepts as 'noun-ness' or 'verb-ness' are built-in by evolutionary processes to the human brain.

Even the early Medieval missionaries who converted Germans and other barbarians were drawn disproportionately from the ranks of the Anglo-Saxon elite and those drawn were highly educated and highly intelligent men such as St Boniface, the Apostle to the Germans; St Martin of Tours was quite unusual in coming from a more common background—his education was minimal and his father was an ordinary Roman soldier.

Christianity is as much a religion of the mind as of the heart and hands and it could easily be argued that that Christian minds need to be well-formed, even those minds of simple believers and not just the minds of potential intellectuals; the minds guides us into the future, powered by the motivations provided by the heart. The mind guides us as we form habits and discipline our selfish, animal desires so that they are shaped to proper moral order.

A partial and distorted understanding, Christianity as doing good and being an example, is more likely to be a stage in departure from Christianity. The mind, individual and communal, which maintains a tie to the creeds of a very complex system of belief but makes sense of science and history and all of that in light of those creeds must be strong if not always subtle or flexible or creative or far-reaching. The communal human mind, if strong and well-formed, can provide much that is needed by simple believers, whether that simplicity is due to some sort of personal choice or to lack of education or to a weak mind.

We modern Christians need to develop words and concepts to build new ways of understanding human nature, life in general, this universe, and all of Creation. In doing so, as Christians, we would generate a moral understanding that would make Creation complete, as a work of the God of Jesus Christ, a work shaped by His purposes for us and all other forms of created being. In doing so, we would make our universe a world, a moral narrative of sorts.

8.2 Making Peace with Empirical Reality

I've used the title of this section before, as a classification of some of the essays published on my blogs, mostly *Acts of Being* [8] but also *To See a World in a Grain of Sand* [9], as republished in a major collection I've updated annually: *Acts of Being* [12]. It is an expression of a major theme in my worldview:

> We human beings, Christians in particular need to take God's Creation more seriously as a revelation and a collection of revelations. We need to give up idealistic systems of thought which no longer match what we know of God's Creation. We need to accept the facts of this world and of all Creation and also need to accept or honestly argue against plausible speculations of all sorts so long as they are based upon reality.

I've also made available a sample of essays on this theme: making peace with empirical reality. See *A Modern View of Creation: Making Peace with Empirical Reality* [11].

8.3 The Human Mind: Individual and Communal and Other

It is important to remember that I am not claiming to come to an understanding of human being which will dictate specific solutions to our problems, many of which have come from the decay of a substantial amount of our human being: individual human being has decayed as a result of cultural processes which have deformed or stunted maturing processes of individuals and communal human being has also decayed badly in many ways.

Roughly, I am trying to provide ways of speaking in more exact terms, properly rich and complex terms, about human being. This forces me to deal somewhat with created being in general and, if only implicitly, with the Being of the Triune God. But the main thrust is to prepare Christian minds to deal with the reality of that communal human being which is necessary to raise us above the level of human animals capable of little more than small-group life, surviving by hunting and gathering and maybe some semi-nomadic type farming. I've just put forward some circular reasoning, but it's the way the world works. Human beings came together and leaders received reproductive advantages because of their social and logistical and fighting skills. Tool-makers and other craftsmen and musicians received reproductive advantages because of their particular types of intelligence and hand-eye coordination and so forth. The more complex skills and lore grew for a group of human animals, the more was required for leaders and others, and so it was that those who had the talent for that more and

developed the talent for that more gained greater and greater reproductive advantages.

The interested reader might wish to start dealing with the issue of human evolution by a reading of the book *The 10,000 Year Explosion: How Civilization Accelerated Human Evolution* [5] by Gregory Cochran and Henry Harpending. In addition to a number of books on evolution and history of ancient peoples, there is a large amount of freely available material on the Internet which deals with such matters as the complex genetic histories of the European peoples, and the rapid evolution of higher IQs in the communities of the Ashkenazi Jews.

So it is that I have modest goals in this book: to provide concepts and some tools for organizing descriptions, for developing deeper and richer and more complex understandings of human being in particular, created being in general.

The images and relationships I use in this discussion are simplistic, though—I believe—not in a way that distorts the underlying reality. In fact, this discussion could be compared to the early stages by which various fields and subfields of modern science and mathematics have developed.

Yet, it isn't the case that I'm really providing a set of 'value-free' techniques. Neither are many who do various sorts of 'modern' analyses of human being, perhaps specializing in one component such as economics or politics or movements of peoples. One common box of tools for these modern analyses are those of a number-crunching flavor, perhaps statistical analysis or perhaps mathematical modeling.

Neither statistical tests nor mathematical models are neutral in terms of assumptions about the real world. Experienced analysts of high professional moral standards can probe data from the world and see what statistical or probabilistic processes are to be—possibly—found in the real world entities producing that data; similarly they can determine what mathematical models might be—possibly—indicated by that data. Most analysts, even if aiming at high professional moral standards, bring with them a tool-box of probabilistic or statistical methods as well as a perhaps similar tool-box of mathematical modeling methods.

Various sorts of scientists who deal with groups as well as individuals have tools for exploring data and putting those individuals in groups—geneticists working on the origins of specific human populations are a good example. There are various websites, as well as typically demanding books, which discuss the general issues and perhaps some of the difficult mathe-

matics and statistical methods used in modern population genetics. The reader, at least one with a sense of humor, might draw something useful from a short entry posted by Gregory Cochran on 2016/09/27: *Super-Gaels* at `https://westhunt.wordpress.com/2016/09/27/super-gaels/`.

Unfortunately, there are many analysts not so sophisticated as Professor Cochran. There are also many who are so sophisticated—peer-reviewed papers in this field and related fields are generally of very high quality, so far as this amateur can tell.

I'll diverge just a little to tell of a personal experience. I did a small project for a friend in an earlier period of my life—she was producing reports for a group of medical professionals who were doing training in some methods associated with a new medical technology. She asked me to do some computer spread-sheets and graphs and also to run some standard statistical tests on surveys of those who had taken the training. I told her to advise the people that those tests (don't remember which letter: t or F or ξ or some Egyptian hieroglyphic or whatever) were for tightly controlled, at least in principle, experiments, such as plants grown under different conditions. They got mad at her and said she was paid to run the tests, not to give advice.

The most impressive of statistical tests or mathematical models produce only garbage if used when their underlying assumptions don't match the situation where they're deployed. From my limited readings, it seems that professional mathematical modelers who work in various fields of engineering, hard science, or softer science don't expect any model to stand up to reality forever, not even the sophisticated models built up by professional modelers. A model works until it doesn't. My worldview seems to me to match what we know of Creation pretty well, but one day we'll know more; before that day comes, someone might demonstrate that my worldview is defective or otherwise inadequate. I hope so.

Reality intrudes and disrupts models, even some of the most powerful and successful models of hard science but also theological models of Creation or philosophical models of created being. The so-called standard model of particle physics was strengthened by the discovery of the *Higgs Boson*—see `https://en.wikipedia.org/wiki/Higgs_boson`. At the same time, the standard model is in trouble because it's known to be incomplete and yet there are no hints in the copious data generated by the experiments at *Large*

Hadron Collider [2] of any sorts of "new physics" in the form of new particles of a known type or—maybe—far more exotic form. Maybe it's the time for an Einstein to think truly new thoughts.

As a sad matter, it's possible the German mathematician Riemann, who worked also in physics as did many mathematicians of earlier centuries, would have developed the theory of General Relativity, perhaps without even going through the indirect path of Special Relativity. Riemann developed the mathematical ideas used by Einstein in General Relativity but died at the age of (almost) 40 in 1866. (See `https://en.wikipedia.org/wiki/Bernhard_Riemann` for a summary biography concentrating on his remarkable accomplishments.)

That thought—of a new Einstein, though not new to me or to mankind, leads me to return to the main line of my work, human being. So far as I can tell, there are no social scientists or philosophers or others who have done what Einstein did when he took on the task of extending the relativistic ideas so successful with electromagnetism to gravity. There is a need to do something similar if we are to come to understand human being in its entirety.

[2] The same facility where the Higgs Boson was discovered.

9 Some More Mathematics Useful for Our Task

9.1 Some Preliminary Mathematical Concepts

It's time to return to mathematics, but not to worry. Not only will there be no tests, there will also be no equations to solve. We will be dealing in concepts, though using definitions, both qualitative and (partially) rigorous. We will be "unscrupulous opportunists," to unscrupulously and opportunistically steal a term from Einstein and to use it slightly differently than he did [1].

I'm taking concepts in order to follow them to a higher level of abstraction and then to follow them down again along slightly different pathways than those which lead to mathematical systems of thought—see Figure 7.1, *Simple Relationships of Abstract and Concrete Levels of Being* for a graphical representation of this sort of movement of mind and imagination.

The highly regarded 19th century mathematician, Charles Dodgson—see `https://en.wikipedia.org/wiki/Lewis_Carroll`, put these words in the mouth of a memorable literary character:

[1] See `https://en.wikiquote.org/wiki/Albert_Einstein` for the full quote. Einstein thought human ideas could be truly 'ideal', that is, free of direct dependence upon empirical reality. I claim that we are creatures embedded in a Creation and can know nothing and think nothing that doesn't, in some strong sense, come from that Creation. On the other hand, I think Creation has multiple realms and some of those realms are far more abstract than the thing-like being of "empirical reality" as defined in narrow terms. Even then, for us to even suspect the existence of those more abstract realms of created being, they would have to have some sort of a presence in our concrete, thing-like universe.

> "When I use a word," Humpty Dumpty said in rather a scornful tone, "it means just what I choose it to mean—neither more nor less." [Lewis Carroll, retrieved from `https://www.brainyquote.com/quotes/quotes/l/lewiscarro389152.html`.]

So long as certain 'rules' (of judgment) are followed, there is no problem with this sort of unscrupulous opportunism. Mathematicians have done it often in recent decades, abstracting from certain relationships in geometry or topology or other fields to find abstract relationships which can be handled by the methods of modern abstract algebra.

'Abstract'.

'Relationships'.

So it is that the mathematically inclined Humpty Dumpty would be right to say that a space is whatever he wishes it to be, so long as his wishes are vaguely coherent, so long as those wishes move towards greater coherence as a result of studying either the abstractions of 'space' or the concrete being towards which those spaces might be applied. In the early stages of the development of modern science and mathematics, the word 'space' meant "Euclidean," rigid and flat.

In more recent generations, the concept of 'space' can take a variety of common-sensical or weird forms, as discussed in Section 2.3. In his second quote on page 18, Aleksandrov tells us that a space is "an arbitrary collection of homogeneous objects (events, states, functions, figures, values of variables, etc.) between which there are relationships similar to the usual spatial relations (continuity, distance, etc.)." In fact, mathematicians have abstracted spaces still further so that there might not be any of these "usual spatial relations," but perhaps only a partial ordering. And maybe not even that.

In Part I, *Laying Some Foundations*, the reader encountered some conceptually difficult material, especially difficult for those who didn't cover at least a basic 2-year mathematics sequence at the college level. Don't worry about it. There is no magic that all thinkers need to possess that they might use the knowledge in one or more human communal minds. When the world presents itself as if having a base set of truths, call it A, then each individual truth, α or β, which belongs to A is what it is. Yet, the human tendency is to explain those irreducible truths, α or β or..., in terms of human schemes built from the some a's and b's read out as irreducible

truths by past generations of men. Any human schemes, even if built with great fidelity to truths as best known at the time, will be very fragile over time—they will fail.

Am I just contemplating my knowledge of the empirical world and of the revelations which come to us through that world and drawing out another set of $a, b, \ldots$ which are no better than an approximation to the 'true' set of $\alpha, \beta, \ldots$?

Not really. In fact, I'm not explicitly pointing to any set of truths which approximate, or point toward, the 'real' set of truths which God manifested as the raw stuff of Creation, the raw stuff of both stuff-proper, concrete and thing-like being, and the raw stuff of abstractions such as lower and higher mathematical truths or the moral truths which give a narrative order to our universe and make it a world.

What I have done is to provide a general description of Creation as being shaped from some truths manifested by God as the raw stuff of that Creation. Clearly, I rather strongly favor a set of manifested truths which are those we Christians see as Being which eventually came to us as Jesus of Nazareth, the human nature of the Son of God.

With some such set of general principles as I am developing explicitly in this book, a thinker can—if so inclined, make more disciplined sense of the ideas in my writings. I'd be the first to admit that my ideas, while inspired and shaped by much modern knowledge in mathematics and physics as well as history and literature, came gushing out before I was able to understand my own ideas to an extent that I could start seeing them as part of a worldview. I'd developed some pieces of that worldview years ago but a box of screws doesn't really give much of an idea of the house it will help to hold together. But it does say something about the other building materials as well as the building methods.

To summarize: I've proposed my take on **some** of the bits and pieces of truths, the most fundamental truths I call the raw stuff of Creation as well the conditioned truths of higher level being which are the results of various shaping processes—evolutionary and developmental—as Creation surged up to this concrete world which was to be and is the mortal home of Jesus Christ, the Son of God in His human nature as well as His divine nature, and is the Home of all that was shaped to provide environments for that incarnate Son of God. That means those are also the environments in which evolve and develop the entire Body of Christ, Jesus Christ and all His friends, the complete man as St Paul presciently realized though he

could not have possibly understood in a way as deep as is now possible by proper use of modern knowledge, abstract but also concrete or empirical.

9.2 Some Thoughts on the History of Abstract Reasoning

In other writings, I've explored an idea which I didn't originate: abstract reasoning ability showed up in certain widely distributed societies in Eurasia with a extraordinary, almost shocking, speed. In an American instant as it were. See *The Origins of European Thought About the Body, the Mind, the Soul, the World, Time, and Fate* [23], *Body, Soul, Spirit: A Survey of the Body-Mind Problem* [28] and *The Discovery of the Mind in Greek Philosophy and Literature* [26] for background on early human abstract reasoning; the last book, *The Discovery of the Mind...*, was a more explicit effort to understand the sudden appearance of human reasoning in one remarkable people. Those particular books are Eurocentric, but the phenomena were more widespread and books, as well as downloadable writings on the Internet, are available which discuss such developments in other parts of Eurasia as well as in the Americas. Remember that even when reading the books not dealing with the suddenness of the rise of abstract reasoning: for good and bad, all of the ideas about body, mind, soul, and all the rest arose very quickly. It was as if a highly clever animal had suddenly begun to think about some of the most abstract ideas which have ever occurred to human beings.

It would seem to me that some still evolving ability, located in still evolving regions of the human brain, suddenly developed to a significant extent when some external conditions were met. What conditions? Complex human technology was appearing during the Iron Age (see `https://en.wikipedia.org/wiki/Iron_Age`) as the semi-nomadic tribesmen living on the steppes of what is nowadays Eastern Ukraine (soon to be ???) and parts of southern Russia learned how to move about more freely to extract resources from various regions in that neck of the woods and also learned how to organize military expeditions and fast-moving migrations of fairly large groups of men and women and children. By this plausible telling of the story, the advanced technologies of iron weapons and iron tools and the breeding and use of horses were largely developed to a high level

in those regions thought by later Greeks and others to be thoroughly barbaric. Those advanced technologies, carried by small warrior bands or larger bands of men and women and children, reached Greece and Southwest Asia and Iran and Northern India and led to the development of warrior-ruled towns, some of which became some of the first cities. China was on its own partly isolated path of development, but seems to have been on largely the same schedule, maybe a little faster and maybe a little slower. The main part of the population of native Americans, largely descended from ancestors shared with Indo-Europeans and from ancestors shared with the Han Chinese were about 500 to 1000 years slower than their distant cousins in Eurasia in showing abstract reasoning and building cities.

The writings of the ancient Hebrews tell the story truly. Hunter/gatherers were the main victims of peoples who were some sort of combination of "first-farmers" and "first-warriors." Those criminal founders of so much that we consider to be true human-ness, even moral decency of a 'civilized' sort, were farmers (Cain as one who somehow offered a defective sacrifice and as the brother of a hunter/gatherer who offered a sacrifice pleasing to God) and warriors (Cain seen as the first 'murderer' and the father of ruthless killers and skilled technologists); Cain's descendants were also the builders of the first cities—at least in Southwest Asia, but—in reality—also in Iran and northern India.

We can picture some vast movements of different streams of human beings—vast relative to the population of the entirety of Western Asia. First came young men with horses and also with iron weapons and iron technology in general. Then came iron-armed men with all that technology but also with their families and they came from the same regions as those young men in the warrior bands. Before and after those streams came farmers out of a region centered on northern Syria. Those three populations, two from the same ancestral group, were major ancestors of the people of the period known to historians as the *Axial Age* (see `https://en.wikipedia.org/wiki/Axial_Age`) which ran from about 700BC to about 200BC. By at least least the 19th century, historians had realized that there were changes during the *Axial Age* which were seemingly more remarkable and more difficult to understand than the earlier changes discussed in the previous paragraphs. Those changes included the sudden showing of abstract reasoning.

Let me jump ahead to the centuries when mathematical thought was coming alive in Europe, the late Middle Ages and early Renaissance. It has

been noted by various historians that some skills and knowledge nowadays taught in the early years of education, such as long-division and double-entry bookkeeping and the solving of polynomials, were once considered matters beyond all but well-educated men with the equivalent of PhD's [2]. Scholars have learned how to state that knowledge in ways that were easier to learn; in the terms I proposing: that knowledge was digested and then absorbed into the communal minds of the West. Much knowledge, many skills, many attitudes were so absorbed. And not just the knowledge and skills and attitudes which are still regarded as parts of the domain of abstract reasoning.

You don't need a PhD to appreciate much of what mathematicians have accomplished in recent centuries. You don't even need extremely detailed knowledge to use mathematical knowledge, though you should be very careful indeed and you should realize you do need sophisticated understandings of the basics of any specialized fields you try to use in reasoning about, for example, the way in which a complex human community can be known as a real entity even though the individuals aren't simply absorbed as they might be in a science-fiction horror movie.

Reductionists or reductionistic materialists have their favorite statistical techniques as well as philosophical presuppositions and ways of writing for scientific journals as well as for the popular press; those favorite ways too often assume the stars and planets are what exist and only what truly exist—in opposition to modern physics and mathematics which deal with individual points or fundamental particles as well as manifolds and a spacetime structure which is our gravitationally bound universe. Even when we consider only concrete, thing-like being, complex entities and even most simple entities are what they are but also bundles of concrete being at various scales and of various sorts. The real number line ($\Re$) cannot be built up from points nor can it be separated from the 'larger' sets described by the transfinite numbers (see `https://en.wikipedia.org/wiki/Transfinite_number`). The universe cannot be built up from quarks and photons and other truly fundamental particles nor can it be separated from the entirety of Creation. Both of those complex entities, the real number line ($\Re$) and the universe exist as real entities—not just nominal entities which are only collections of smaller

[2] To be a little picky, some of those high-achieving mathematicians of that period were self-educated or servants of mathematicians

things. But those complex entities also exist as themselves parts of greater wholes, but not as parts of Creation which are only parts.

Reality is altogether more complex than, different from, a building block construction.

Yet, we have to work our way towards partial understandings of that complexity, that totality of parts and totalities.

Let's now turn back to some more discussions of mathematical concepts.

9.3 Sets, Including Some Strange Forms of Mathematical Being

Set In mathematics, a set is a well-defined collection of distinct objects, considered as an object in its own right. For example, the numbers 2, 4, and 6 are distinct objects when considered separately, but when they are considered collectively they form a single set of size three, written $2, 4, 6$. Sets are one of the most fundamental concepts in mathematics. Developed at the end of the 19th century, set theory is now a ubiquitous part of mathematics, and can be used as a foundation from which nearly all of mathematics can be derived. [See *Set(mathematics)* at `https://en.wikipedia.org/wiki/Set_(mathematics)`]

A human being could be defined as a set of relationships which generate and sustain a body of flesh and blood. If we define these relationships in full, concrete detail, we'll have a description of a real human being with much, shall we say, factuality. If we had the power of God, that description could bring into existence a real human being in the flesh.

Somewhat equivalently for an existing concrete entity, we can define the stuff which those relationships generate, largely by manipulating existing concrete stuff such as water and carbon dioxide and various ingested proteins and fats and carbohydrates and so forth. At this level of manipulating concrete elements, we human beings do share quite a bit of God's power to shape entities—at least in principle and maybe before long, we might have such power to shape new life from mere chemicals. Moral philosophers and moral theologians are working on the problems of having such power, though many such thinkers seem fully clueless.

If we take a snapshot, your body as it exists at this instance, we might very well end up with a set of descriptions of each fundamental particle (protons and neutrons made of quarks which are just momentary quantizations of strong nuclear fields, electrons which are momentary quantizations of electromagnetic fields, etc) [3]. See the article *Quantum field theory* at `https://en.wikipedia.org/wiki/Quantum_field_theory` for a brief but necessarily confusing introduction to what seems the real, that is—mathematical and abstract, nature of the particles which make up rocks and rattlesnakes and pretty girls.

For our purposes, all of this elegant stuff is easily seen as one of the greatest accomplishments of the human mind and spirit but it's as eerie as it is interesting to contemplate this ghostly image of concrete things as trillions and trillions of quantizations of still more ghostly fields describable only in very abstract terms, the terms of the—roughly speaking—three formulations of quantum physics—one due to Erwin Schrödinger (see `https://en.wikipedia.org/wiki/Erwin_Schrodinger`), one to Werner Heisenberg (see `https://en.wikipedia.org/wiki/Werner_Heisenberg`), and one to Richard Feynman (see `https://en.wikipedia.org/wiki/Richard_Feynman`). In any case, all this brilliant experimentation and theorizing and speculation so insightful to the nature of some aspects of concrete being in this universe indicates the sheer silliness of any radical reductionistic way of thought, of any attempt to see this world as just its physical stuff playing out a set of actions governed only by physical forces. Some sort of materialism is clearly necessary but there's something else going on other than the mere gathering together of protons and neutrons and electrons and...

If we jump to the concrete level of flesh and blood, we can define a human being—tongue in cheek—as a featherless biped or—in a somewhat less cheeky way—as a political animal or the like. For most purposes, this is a better level of thought, though we must always remember that, as one example, this stuff can exist only because of the strangeness of all that stuff studied by quantum field theory. As one example, a world which operated by the forces of classical ('Newtonian') physics couldn't remain in existence; electrons would lose energy and spiral in towards the nucleus of an atom. The matter of this world would have actually destroyed itself within small

[3] The situation would actually be worse for human understanding as each set which is a description of a human being would be a set of many sets interacting with each other.

fractions of a second after coming into existence.

Let's push most of this to the back of our minds for now and get on with a little more mathematical thought.

First, let's think of the set which is a human being in simple terms, as we did in Chapter 5, *States of Realized or Concrete Being*. As an exercise, we had sort of reduced a human being to a description of his or her height and weight.

Let's complicate and complexify matters a bit, but first let me throw out a definition of an open set, a set of a certain type:

Open set In topology, an open set is an abstract concept generalizing the idea of an open interval in the real line. The simplest example is in metric spaces, where open sets can be defined as those sets which contain an open ball around each of their points (or, equivalently, a set is open if it doesn't contain any of its boundary points); however, an open set, in general, can be very abstract: any collection of sets can be called open, as long as the union of an arbitrary number of open sets is open, the intersection of a finite number of open sets is open, and the space itself is open. These conditions are very loose, and they allow enormous flexibility in the choice of open sets. In the two extremes, every set can be open (called the discrete topology), or no set can be open but the space itself (the indiscrete topology). [See *Open set* at `https://en.wikipedia.org/wiki/Open_set`.]

This intuitive discussion of open set goes on to say:

> In practice, however, open sets are usually chosen to be similar to the open intervals of the real line. The notion of an open set provides a fundamental way to speak of nearness of points in a topological space, without explicitly having a concept of distance defined. Once a choice of open sets is made, the properties of continuity, connectedness, and compactness, which use notions of nearness, can be defined using these open sets.

These sets, in ordinary and imprecise but meaningful words, merge into their surroundings. Think in terms of a disk with no real boundary. There are interior regions, private regions if you will, but there is no firmly defined edge to keep out the exterior regions.

We can go further. There is nothing that says a set has to be like that disk, continuous inside the rim whether closed or open. There is nothing that says sets can't overlap, sometimes in strange and serious ways. But overlapping sets might not really be in contact. Let's consider a very strange set: *Cantor set* at `https://en.wikipedia.org/wiki/Cantor_set`. This set has the same number of members as the set of real numbers but it has zero measure—there is no interval in the Cantor set which has more than one point (to speak in a true but somewhat sloppy way). If you page down a little in that article (or go directly to `https://en.wikipedia.org/wiki/Cantor_set#Cantor_dust`), you can read:

> Cantor dust is a multi-dimensional version of the Cantor set. It can be formed by taking a finite Cartesian product of the Cantor set with itself, making it a Cantor space. Like the Cantor set, Cantor dust has zero measure.

Before going on, I'll provide an informal, but accurate definition for Cartesian product:

Cartesian product A mathematical operation that returns a set (or product set or simply product) from multiple sets. That is, for sets A and B, the Cartesian product A ÃŮ B is the set of all ordered pairs (a, b) where $a \in A$ and $b \in B$. Products can be specified using set-builder notation, e.g.

$A \times B = \{(a, b) \mid a \in A, b \in B\}$

More generally, a Cartesian product of n sets, also known as an n-fold Cartesian product, can be represented by an array of n dimensions, where each element is an n-tuple. An ordered pair is a 2-tuple or couple.

The Cartesian product is named after René Descartes, whose formulation of analytic geometry gave rise to the concept, which is further generalized in terms of direct product. [See `https://en.wikipedia.org/wiki/Cartesian_product`.]

Let's take a simple example. If $A = \{0, 1\}$ and $B = \{2, 3\}$, then the Cartesian products would be $(0, 2), (0, 3), (1, 2)$, and $(1, 3)$. A simple but powerful way of building a specific type of set useful in many tasks in

abstract mathematics and in a variety of applied fields. So it is that you can construct, rigorously, a set in 3 or more dimensions which is infinitely fine—infinitely dusty—and yet has an uncountably infinite number of members.

Multiple such strange sets—'strange' by the standards of we who live in this world in which we perceive directly, or naively or whateverly, concrete thing-like being—might well exist in the same regions of the same space without having a single point in common, even though both sets might have an uncountably infinite number of members.

I'll try to draw some lessons from this subsection before moving on.

Set theory can be used to describe collections such as those of even numbers and numbers evenly divisible by three, so that we can easily see three sets:

1. $A = \{2, 4, 6, 8, \ldots\}$,
2. $B = \{3, 6, 9, 12, \ldots\}$, and
3. $C = \{6, 12, 18, 24, \ldots\}$.

Obviously, I've restricted the example to positive integers, those greater than 0.

The set, or collection, C is the intersection of A and B, the set of numbers which are even (evenly divisible by 2) and also evenly divisible by 3. C is also seen to be the set of numbers which are evenly divisible by 6 or 2×3.

Most importantly for my current purposes, such ways of thinking can free the imagination in certain ways as well as subjecting that freed imagination to certain types of discipline. Such ways of thinking are to be encouraged in children because it is one of those early steps in the development of mathematical knowledge and skills which is both simple (to some) and an act of true insight or genius.

9.4 Topological Spaces Again, Sort of

In Part I, *Laying Some Foundations*, I wrote a little bit vaguely about topology, emphasizing that it is (partly) qualitative and its role in abstracting of abstractions (expansion of freshman calculus 'limits' to sets in which there isn't even any measurement of distance between members and so on).

Let's get a bit closer to a tighter understanding of topology for the purpose of understanding the nature of modern mathematical thought in general:

Topology In mathematics, topology (from the Greek [topos for] place, and [logos for] study), is concerned with the properties of space that are preserved under continuous deformations, such as stretching and bending, but not tearing or gluing. This can be studied by considering a collection of subsets, called open sets, that satisfy certain properties, turning the given set into what is known as a topological space. Important topological properties include connectedness and compactness.

Topology developed as a field of study out of geometry and set theory, through analysis of such concepts as space, dimension, and transformation. [See *Topology* at `https://en.wikipedia.org/wiki/Topology`.]

That same Wikipedia article tells us:

> Topology can be formally defined as "the study of qualitative properties of certain objects (called topological spaces) that are invariant under a certain kind of transformation (called a continuous map), especially those properties that are invariant under a certain kind of transformation (called homeomorphism)."
>
> Topology is also used to refer to a structure imposed upon a set X, a structure that essentially 'characterizes' the set X as a topological space by taking proper care of properties such as convergence, connectedness and continuity, upon transformation.
>
> Topological spaces show up naturally in almost every branch of mathematics. This has made topology one of the great unifying ideas of mathematics.
>
> The motivating insight behind topology is that some geometric problems depend not on the exact shape of the objects involved, but rather on the way they are put together. For example, the square and the circle have many properties in common: they are both one dimensional objects (from a topological point of view) and both separate the plane into two

parts, the part inside and the part outside. [See *Topology* at `https://en.wikipedia.org/wiki/Topology`.]

In Section 9.3 above, *Sets, Including Some Strange Forms of Mathematical Being*, we saw that open sets can be strange objects, yet, they are used to define the fundamental ideas of topology. Most readers should move on without worrying too much about the use of open sets, but I'll reinforce the idea in a quotation found in Part I, *Laying Some Foundations*. That quote on page 28 supports one of my claims: that even abstractions are drawn from the real world, from experience and experimentation. Basic books on topology, such as *Topology* [18], will tell the reader that it was experience with various ways of defining the fundamental ideas of topology, or other similar concepts, that led to the use of open sets in defining 'topological space' in terms of open sets.

Let's move on to get a little closer to a rigorous definition of topology:

Topological space In topology and related branches of mathematics, a topological space may be defined as a set of points, along with a set of neighbourhoods for each point, that satisfy a set of axioms relating points and neighbourhoods. The definition of a topological space relies only upon set theory and is the most general notion of a mathematical space that allows for the definition of concepts such as continuity, connectedness, and convergence. Other spaces, such as manifolds and metric spaces, are specializations of topological spaces with extra structures or constraints. Being so general, topological spaces are a central unifying notion and appear in virtually every branch of modern mathematics. The branch of mathematics that studies topological spaces in their own right is called point-set topology or general topology. [See *Topological space* at `https://en.wikipedia.org/wiki/Topological_space`.]

This is a more truly rigorous definition of 'topological space' in terms of those mysteriously important 'open sets':

Topological space (in terms of open sets) A topological space is then a set X together with a collection of subsets of X, called open sets and satisfying the following axioms:

1. The empty set and X itself are open.

2. Any union of open sets is open.
3. The intersection of any finite number of open sets is open.

The collection τ of open sets is then also called a topology on X, or, if more precision is needed, an open set topology. The sets in τ are called the open sets, and their complements in X are called closed sets. A subset of X may be neither closed nor open, either closed or open, or both. A set that is both closed and open is called a clopen set. [See *Topological space* at `https://en.wikipedia.org/wiki/Topological_space`.]

As an act of mercy to me as well as the reader, we will ignore the concept of 'clopen' for the rest of the book.

In *Topology* [18], we are told on page 5, that:

> A point p is a limit point of a subset X of S provided that every open set containing p also contains a point of X distinct from p.

Limit point [A] limit point of a set S in a topological space X is a point x (which is in X, but not necessarily in S) that can be "approximated" by points of S in the sense that every neighbourhood of x with respect to the topology on X also contains a point of S other than x itself. Note that x does not have to be an element of S. This concept profitably generalizes the notion of a limit and is the underpinning of concepts such as closed set and topological closure. Indeed, a set is closed if and only if it contains all of its limit points, and the topological closure operation can be thought of as an operation that enriches a set by uniting it with its limit points.

What and where are these limit points? Let me throw out an informal definition of a specific topology which will be discussed in the early pages of any modern textbook on general or point-set topology—the one I refer to for deeper understanding and general principles, *Topology* [18] by Hocking and Young, covers both that sort of viewpoint and also the more 'up-to-date' viewpoint of algebraic topology, a more elegant and more austere viewpoint indeed.

Let me introduce a couple of specific topologies, think of them in terms of the real number line, $\Re$, or the Cartesian grid (two-dimensional real

number plane familiar from simple geometry), $\Re^2$. Both of these topologies, the trivial topology and the discrete topology, are introduced in the early chapters of any textbook on general topology.

Trivial topology [A] topological space with the trivial topology is one where the only open sets are the empty set and the entire space. Such a space is sometimes called an indiscrete space. Intuitively, this has the consequence that all points of the space are "lumped together" and cannot be distinguished by topological means; it belongs to a pseudometric space in which the distance between any two points is zero. [`https://en.wikipedia.org/wiki/Trivial_topology`]

There is zero distance between points and the entire set S is homogeneous in the sense that points can't be distinguished by topological means. Analogically, this can be seen as a sort of hive, a Borg hive for most modern people, an even more extreme version of a Stalinism, where in principle, only one man—the king-bee—was free though he was enslaved by his own paranoia and perhaps by his sadism. In any case, those not up to date can see `https://en.wikipedia.org/wiki/Borg_(Star_Trek)` for more information. In more explicitly rational terms, this topology is analogical to community as an absorbing entity which destroys all the independent existence of its members or at least the ability to talk or think about that independent existence.

Discrete topology [A] discrete space is a particularly simple example of a topological space or similar structure, one in which the points form a discontinuous sequence, meaning they are isolated from each other in a certain sense. The discrete topology is the finest topology that can be given on a set, i.e., it defines all subsets as open sets. In particular, each singleton is an open set in the discrete topology. [`https://en.wikipedia.org/wiki/Discrete_space`]

More simply, each point is isolated. This is analogical to a situation where only the individuals, point-like, have true existence and any groupings have only nominal existence. Note: individuals don't even cover a region; if they did, overlap would be possible. Analogically, this can be seen as freestanding individuals, not interpenetrating each other.

What meaning can be drawn from the two topologies discussed above? I'm actually not suggesting we try to use this discussion directly in understanding human being, individual and communal. Rather am I suggesting we should stand back and realize what is going on. Two different topologies, trivial and discrete, can be applied to a set, say that of real numbers, to produce radically different mathematical landscapes. Analogously, descriptive and qualitative 'topologies' can be applied to human being to produce radically different human landscapes: one of a totalitarian communism and one of radical individualists not inherently connected to one another. I'm further suggesting a descriptive and qualitative 'topology' can be developed which is between, in some sense, the two extremes. The resulting human landscape would be populated by individuals who also share in communal being, at a variety of levels and involving a variety of communities religious and political and professional and so on.

We need to abstract from topological space to spaces of similar characteristics which might include the particular subspaces which are topological (mathematically rigorous) spaces and also some subspaces and an inclusive space which have more abstract, more general, less rigorous characteristics. It's already possible to construct topological spaces which have no way of measuring distance, as in the trivial topology above where "the distance between any two points is zero." We can have sets with no metric (way of measuring distance) but some way of putting elements into some sort of order. We can have all sorts of strange spaces. It might even be the case that modern mathematics has all the abstractions necessary to provide descriptions of all parts and aspects of human being as it is in this mortal realm.

Let me emphasize:

> Use of mathematical concepts and tools doesn't mean that we treat human being and other more general forms of created being as if they were **only** physical matter.

Let's explore a little. Let's be child-like as Jesus Christ Himself advocated. Let's be curious and openminded, willing to learn from the human beings around us and from our dogs and the materials we use to make tables or the abstractions we use to make pretty images on our computer screen. We should be willing to learn from the physical universe being explored by astronomers and cosmological physicists, from playful poets and musicians

blending African rhythms and Celtic melodies, from dancers and graceful outfielders. We should learn from architects and house-builders, cabinet-makers and roofers, geologists and volcanologists, oceanographers and planetologists, historians and brain-scientists, parents and teachers and priests and ministers and rabbis, as well as the mathematicians and other scientists who are the explorers and analysts of the abstract being of primary interest in this book. We learn how to abstract from the properly rigorous concepts and methods of mathematics by experimentation probably taking at least a few generations to settle down into a more orderly scheme.

9.5 Let's Picture a Simplified Situation

A picture can sometimes help, though I'll provide only pictures which are similar to those found in the early chapters of an introductory book on topology or differential geometry. There will be no fancy pictures, only simple pictures which help support new ways to think about human being, created being in general. Before moving on, some readers might wish to quickly review the definition of homeomorphsism—see page 51.

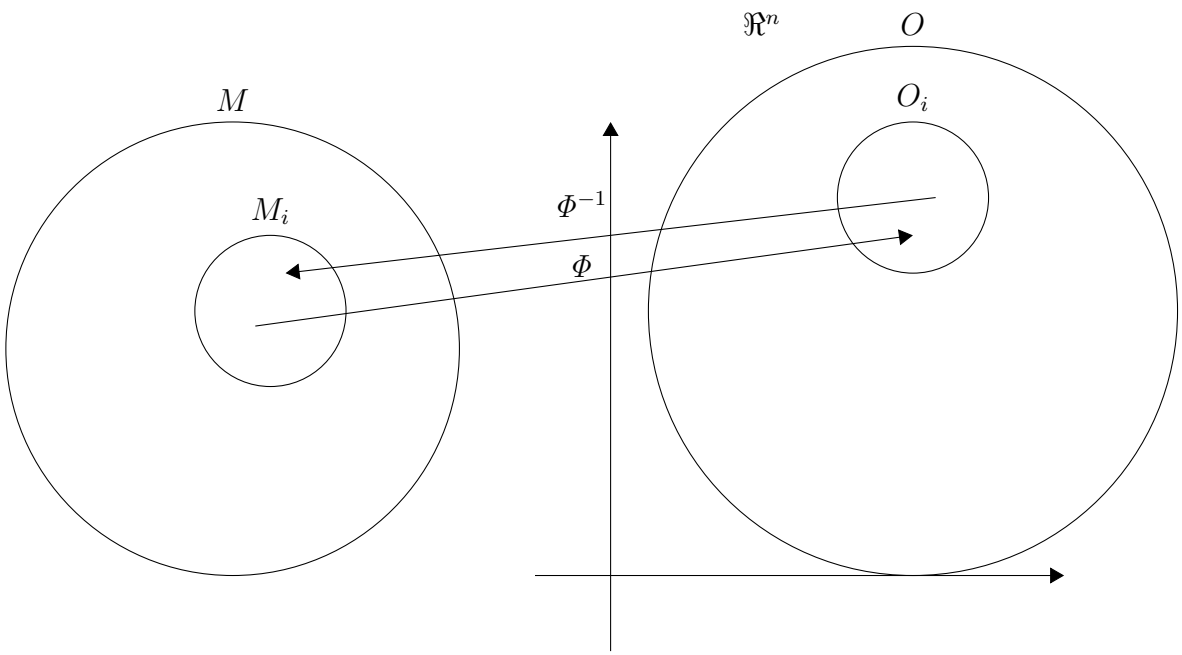

Figure 9.1: A Basic Homeomorphism

;

What does this graph represent? Or maybe: What is it supposed to represent? The graph will be as obvious to those who have taken a basic topology course or, probably, to those who have taken an analysis course—think of analysis as being approximately advanced or sophisticated calculus. For others: I'll try to explain the above graph in simple terms.

Don't take the circles literally, they represent general spaces. The circle at the left, M is a manifold which is being mapped to O which is a Euclidean space. For intuitive understanding, think of M_i as a region on the

curved surface of the Earth and O_i as a flattened map of that region, by way of a transformation such as the *Mercator projection* which is described at `https://en.wikipedia.org/wiki/Mercator_projection`. This particular projection is the one which is used to generate most flat maps of large regions of the Earth.

With that in mind, I'll provide short descriptions of the components of the graph, that is, of the named variables:

$\Re^n$ The real number line (n=1), plane or 2-dimensional grid of two real number lines, such as an x-y graph (n=2). More generally, the n-dimensional real number coordinate system where a point would be $(x_1, x_2, x_3, \ldots, x_n)$.

M The manifold (see page 51 of 'points' which is locally Euclidean (or locally resembles a Euclidean space) and is mappable in a useful way to a Euclidean space. Moving beyond the simple case of maps of the Earth's surface—which is the manifold and not Euclidean, these points can be thought of as 'brain' but, more truly, as all parts of human physical stuff which contributes to the mind of that individual. That means the entirety of the embodied human being, though not all parts of the human 'body' contribute equally to the mind. The brain is obviously dominant.

O A Euclidean space, $\Re^n$, corresponding to all members of the community of M and also to the shared or communal minds of that community. Take some of this as only vaguely or intuitively defined for now, but the general idea is that this is a space which is globally Euclidean, like those spaces of high school courses in plane geometry or calculus.

M_i A specific small region of M, such as a member of the community which is described by M. Each subscript i=1,2,3,... represents a individual's physical stuff which contributes to mind. When it comes to cognitive aspects of a human being, it would seem useful—perhaps only as a first step—to think of this as the brain of the physical human animal.

O_i A Euclidean space to which a specific small region of M_i is mapped.

Φ The homeomorphic function which maps each region of M to O.

Φ^{-1} The reverse homeomorphic function which maps each region of O back to M.

In modernist discussions of politics as one example, this space, O, would be the collection of individuals corresponding to modern assumptions of freestanding individuals. There can be only some very small overlapping in such maps but perhaps none at all—a libertarian man might be an island. The overlapping would correspond to such things as thoughts shared in some sort of community or other group (such as widely scattered viewers of Hollywood movies). It can also correspond to shared feelings or attitudes as well as shared ways of behavior.

In other words, O can be defined as the space analogical to the false view of human being held by most modern thinkers in politics and other social sciences, by some modern men of literature, by most politicians and Christian leaders, and by seemingly most modern human beings in the West. Even traditionalists who try to recognize the reality of communal human being, think and speak and write far too often in these terms because they have no explicit models of the sort I'm developing here: models in which individuals are treated as such but are always tied back to a more complex reality of individuals and communities. The simpler models are truly useful and can be used to develop and explain many aspects of human being. They don't reflect the fullness of truth which is found in the complex geometry of the manifold M.

Notice something very important. There is not only a function Φ which maps small regions of the manifold M onto regions of the Euclidean space O; there is also an inverse function Φ^{-1} which maps those regions of the Euclidean space O back to the original small region of M.

Before I move on in this discussion of a simple model, I have to introduce the reader to a new concept in topology as well as a specialized version of that concept.

Connected space a connected space is a topological space that cannot be represented as the union of two or more disjoint nonempty open subsets. Connectedness is one of the principal topological properties that are used to distinguish topological spaces. [`https://en.wikipedia.org/wiki/Connected_space`]

Path-connected space A stronger notion [than connected space] is that of a path-connected space, which is a space where any two points can be joined by a path.

In *Rewriting the Soul: Multiple Personality and the Sciences of Memory* [16], the philosopher Ian Hacking tells of his work with victims of serial rapists or (would-be) killers and his consequent meeting of some jailed criminals of that sort. During this work, he formulated an hypothesis that multiple personalities (which are disproportionately common among victims and maybe among some of those sorts of serial abusers.) He speculated that the problem is in the memory: walls of a sort develop somehow to sequester very bad memories. We need work in this area and any good and insightful work would have to be based upon a more plausible understanding of human being. I think my way of approaching these sorts of problems would provide such an understanding.

We should maybe think of unity of a human animal in terms of path connectedness. Are all the parts and aspects of that human animal reachable by a direct path? This might involve travel through a specific organ or sub-organ, such as the hippocampus so important in human memory formation and recall. Even cognition or processing of perceptions might involve some paths through glands or other parts of the body outside of the brain. Path connectedness of this sort of communication and information processing network might very well involve multiple paths, so long as you get from any point A to any point B in that human animal.

In my readings in accessible books on brain-science, mostly written by highly regarded researchers such as Larry Squires and Gerald M Edelman and Michael Gazzaniga, I get the impression that the thinkers in this field dominated by some extremely smart human beings, do think in these sorts of terms and I'd guess that some use topological concepts and knowledge directly, but that doesn't help in the general spread of high-level knowledge into the communal minds—that sort of thinking has to become more explicit in the way that, for example, body-mind dualism became part of our communal minds by way of ancient theories (sometimes theological or purely speculative) but also by way of the work of thinkers such as René Descartes, a man certainly at least as great a scientist as any working in modern brain-science. Many popular works on the human brain-mind do an awful lot of handwaving as do even some profound philosophical and theological works of the modern era. While

there have been some advocates of dualism, most notably Stanley Jaki (see `https://en.wikipedia.org/wiki/Stanley_Jaki`), who are clearheaded, even they and even-even the opponents of dualism can't quite produce models which describe the human mind, either as something (partly) different from the brain or as a set of relationships formed by a human being, individual or communal, with his or its own self and with all in Creation to which that individual or community responds.

I'll move on to suggest that the region which is communal mind is path-connected to every region which is an individual mind. This means that every mind in this community which is path-connected to every other mind, that is, there are paths connecting every mind which run **through the communal mind**. These communal path-connections are relationships between relationships, even at the foundation, rather than being founded upon relationships between neurons and glands and other body parts. Individual minds (which are sets of relationships) will be connected directly by one or more paths, maybe many paths, just as individual hearts (feelings) and individual pairs of hands (habits and other behaviors) will be connected directly many times over to other individual hearts and individual pairs of hands—but not likely to all individuals in this community.

As a rule, we will form more explicit connections to minds 'close' to us rather than only connecting through the communal mind. This might mean the physical closeness of household members or neighbors or local communities of worship. It might mean the different closeness of professional societies or sports clubs or academic fields.

The concepts I'm advocating can be used for analyses in which an individual is taken out of his community by a mapping which is a flattening, a removal of the curvature which makes him part of a community. This is often proper and often does not even a little damage to the facts of human being, communal as well as individual.

9.6 Let's Get Just a Little More Realistic

Once again: I'm trying to develop a box of conceptual tools by dealing with the more or less specific task of talking about human communal beings, communities, as real and not just ways of speaking about collections of individual human beings.

The graph below presents a slightly enhanced version of the graph above. Some recognition has been added of the communal component of the individual (small region) on the manifold.

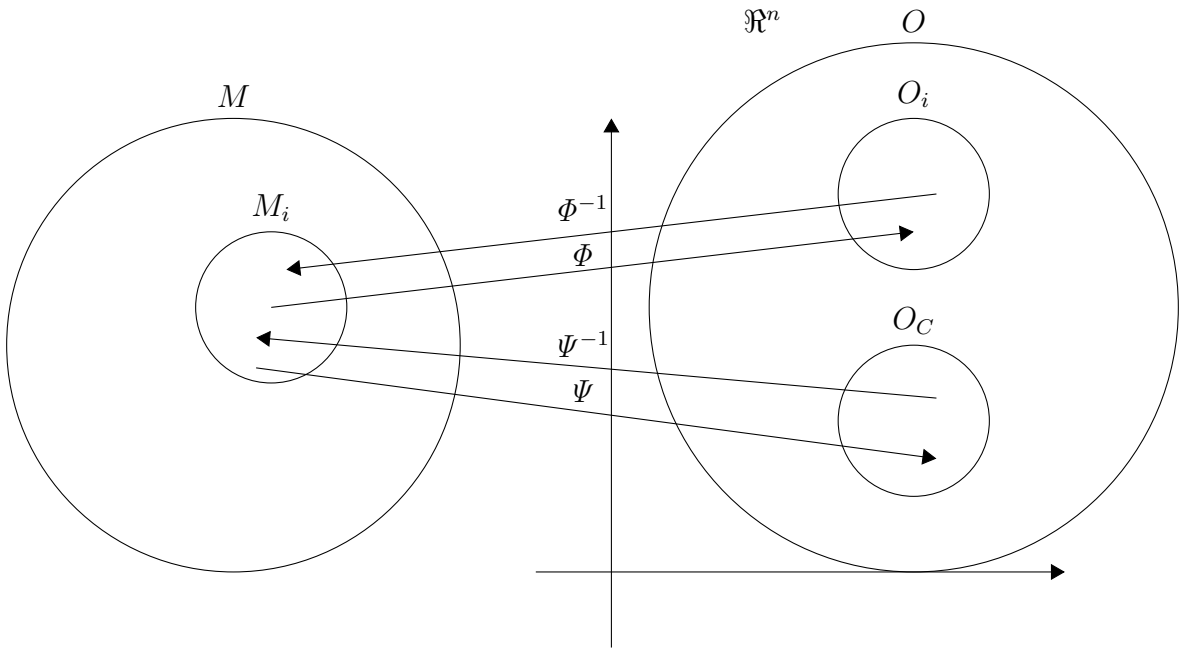

Figure 9.2: Quite Simplistically: Individual and Communal Mind

;

The variables added to this figure:

O_C A Euclidean space to which all shared or communal elements of the community mind of M_i's are mapped.

- Ψ The homeomorphic function which maps each region of the shared or communal mind components of M to O. Many of these components are located in the individual minds. In my simplistic figure, all of them are so located.

- Ψ^{-1} The reverse homeomorphic function which maps each region of the shared or communal mind components of O back to M.

In thinking about O_C, we should think not in terms of a separate region within the space of $\Re^n$. Though I'm not fully confident about this being the best or most accurate representation, I would advocate thinking of the communal mind as an extra n-dimensional layer over (or under) the entire manifold O. There can be, and usually will be, overlap of O_C and the individual regions O_i where i=1,2,3.... This is the overlap discussed in the previous section which described a simpler model.

The model in the above graph, figure 9.2, as well as the model in the graph in the previous section, figure 9.1, are—as noted in the captions—simplistic. They are not just simple, but simplistic and somewhat unreasonable but suitable for purposes of initial exploration of a line of thought. They are based upon examples in the early chapters of *Topology* [18], *Introduction to Global Analysis* [19], and *Topology and Geometry for Physicists* [22]. As is true in those introductory textbooks, I've presented models—or re-presented the models of those textbooks in the context of human being—as teaching aids. And I'm one of my students, the only one that I'm in direct contact with right now.

Part IV

What Have We Done and Where Should We Head?

10 The Knowledge Possessed by a Creature

What is truth? Not so ridiculous a question as some would have it, certainly those who don't understand how difficult a struggle it has been to develop various plausible and implausible understandings of the abstract concepts of 'truth' and 'knowledge' over the past 10,000 years or so of rapid increase of human intelligence in certain Eurasian peoples—see the previously referenced *The 10,000 Year Explosion: How Civilization Accelerated Human Evolution* [5] by Gregory Cochran and Henry Harpending for an introduction to the genetic-historical study of the underlying events. If you wish to read of these events, or at least of snapshots of these events, from a philosophical or historical viewpoint, a viewpoint also more or less Eurocentric, you can see *The Origins of European Thought About the Body, the Mind, the Soul, the World, Time, and Fate* [23] by R B Onians, *Body, Soul, Spirit: A Survey of the Body-Mind Problem* [28] by C A van Peursen, and *The Discovery of the Mind in Greek Philosophy and Literature* [26] by Bruno Snell. General searches on the Internet or in your local library system on the *Axial Age* (see `https://en.wikipedia.org/wiki/Axial_Age`) will lead you to to a variety of 'old-fashioned' and 'new-fangled' historical works on the strange and glorious transformation in human culture, including the intellect, which occurred from some time in the *Iron Age* (see `https://en.wikipedia.org/wiki/Iron_Age`) and ended a few centuries or so before the birth of Jesus Christ. For a historical perspective which relies on particular mathematical and statistical analyses rather than starting at the level of abstract being as I'm doing, see Peter Turchin's website at `http://peterturchin.com/` for an interesting, plausible, and powerful attempt at building applied mathematical models of events in human history; at the least, there are some good insights in the work of Turchin and

allied thinkers. (One of Turchin's non-technical summary of much of his work, *War and Peace and War: The Life Cycles of Imperial Nations* [27] is itself a pretty work of history.)

The writings referenced in the above paragraph are but an entry into various sorts of literature dealing with the results of an important period of evolutionary and developmental changes in human being, changes which led to Archimedes and Augustine and Aquinas and Fra Angelico and Pascal and Newton and Mozart and on to Planck and Einstein and Picasso and Arnold Schoenberg and Eliot and Joyce and Sartre and others. A few minutes of serious thought will lead to the realization that any list of high achievers in recent centuries is remarkably short on great religious thinkers (John Henry Newman was at least a second-tier great thinker but few others reached anywhere near that) and writers or artists working in the traditionalist mode (Eliot was almost entirely modern in style though he advanced—did himself **adhere** to—important, central lines of traditionalist thought). Scientists and various mathematicians, including some great mathematicians discussed in earlier pages of this book, seem to be still more dominant, shockingly dominant, when we limit modern thinkers and writers and artists to those of undoubted greatness.

I'm not sure how many scientists or mathematicians would share my understanding of what-is, merely the universe to many of them and Creation to some of them and certainly to me. Only a few, mostly Platonists and mostly mathematicians, would even grasp the concept of abstractions as a form of being. Probably those Platonists would make the same understandable error as Plato—thinking of abstractions in terms of Reals which seem to be pure and ideal archetypes of complex entities such as human being; this idea seems implausible after Darwin and Einstein as seen through the eyes of the Reverend Monsignor Georges Lemaître (see `https://en.wikipedia.org/wiki/Georges_Lemaitre`) and other founders of modern physical cosmology (in the 1920s).

After the work of evolutionary biology and quantum physics and history, we don't even currently know what are the basic components of being, human or otherwise—where 'know' can be taken as involving the actual truth or just the most plausible speculation of an age. We don't even know if there are basic components of the sort found in particle physics or nuclear physics. I suspect not, having found it easier to make sense of Creation in terms of abstract being and concrete being—to be sure, I think of concrete being as shaped from abstract being so that abstract being is something like

an elementary form of being but it's a bit different and more consistent with quantum physics. In other words, the wavefunctions of quantum physics aren't like the elementary entities of particle physics, though those entities correspond to quantum wavefunctions.

We have no coherent and morally well-ordered understandings of Creation in light of evolutionary biology or quantum physics. Our moral philosophers and moral theologians and various leaders don't know how to view our communities and individuals in terms allowing even potential moral order of a sort which once seemed so clear to our minds and eyes. Our theologians in Christian traditions don't know how to talk about man's meeting with God, in prayer or worship or more specifically in the Sacraments; they babble on using terms which once meant something when the Greek philosophers and Medieval Scholastics seemed to have provided solid understandings of matter and mind and even God [1]. Modern thinkers sometimes just accept schemes of words and concepts of premodern times and sometimes try to build up their own—none of those modern schemes seem to be at all plausible in light of what we now know about Creation.

We are creatures embedded in a Creation arising from raw stuff which is and are the truths manifested by God through His Son. To our minds and hearts and hands, this Creation is quite dynamic for at least three reasons:

- Creation is so because it is inherently so, evolving and developing as it moves from its original primordial state toward greater particularity and complexity.

- Creation is so because we learn about it through our own evolving and developing minds and hearts and hands.

- Creation is so because our very efforts to understand and to exploit this Creation brings about changes in the evolution and development of our own human being and in much of the being with which we can directly form relationships of any sort.

[1] The Protestant Reformers disagreed, sometimes radically so, with the specific, 'higher-order' Catholic schemes for understanding Sacraments and a sacramental world and man's moral situation and so on, but accepted the words and concepts which existed at that time for discussing these things. Neither Martin Luther nor the Catholic Counter-reformer Cardinal Robert Bellarmine even pretended to engage in creative metaphysical or theological thought.

In my updated version of Thomism, Creation is not a place containing entities but rather a set of relationships which are acts-of-being. Material stuff and even abstract stuff is the result of those relationships, most of which are highly dynamic. Human knowledge isn't settled from our viewpoint, human knowledge doesn't encapsulate the truth, until we can reach a better, more stable viewing place. But no such place can possibly exist so long as we're alive because we learn more about Creation and we also change what is by our active responses, as do all entities and lesser creatures, but only we humans can—in principle—encapsulate all that God has created in our own minds. Only we humans can—in principle—participate in God's greater acts-of-being, His thoughts and feelings and actions which take place in any realm or all realms of Creation. Only we humans can—in principle—share God's thoughts even as He creates from nothing.

Some principles can only be realized in the world of the resurrected where the friends of God share God's life by being part of the Body of Christ, by sharing directly in the human being of the Son of God. By so sharing we might even enjoy the dizzying sight of Creation from God's transcendental viewpoint. Then we will be able to understand.

11 From the Ideological Frying-pan to the Ideological Fire

From 2006 to the present, I've been writing about the need to make peace with empirical reality. This is not simply a matter of a one-time adjustment when reality reveals itself as different from, usually richer and more complex than, the inherited worldview of a particular human being—individual or communal.

Unfortunately, it turns out that most human beings aren't very good at making even limited adjustments in their understanding of their environments, however defined. Even most of those with minds capable of learning difficult material and difficult ways of handling that material are not really very good after their initial education or training at handling material, difficult or easy. If new knowledge doesn't fit into the slots formed in their minds during their youth, it is often ignored even at the cost of great incoherence.

So, what happens when a culture—far worse, an entire civilization—falls into a disorder connected to—perhaps even partly caused by—a collapse of the communal mind of that culture or civilization? We need new thoughts or new ways of behavior and even new attitudes but all we get from the intellectual and religious and cultural and political leaders of the West is great confusion and little in the way of plausible suggestions for fixing our problems so that we can avoid the collapse of the West. Of course, there are some in those leadership categories who aren't inclined to respond properly since they're busy looting the resources under their stewardship; we'll ignore those in this discussion.

There are various short analyses and short comments I've read about this situation: the collapse of a civilization and the general failure to do what's necessary to save it or at least to pave a reasonably smooth road

into a new civilization. In *The Evolution of Civilizations: An Introduction to Historical Analysis* [25], Carroll Quigley wrote about the formation of human instruments of a new or reformed civilization; over time, these instruments become self-serving human institutions which will struggle, through institutional means as well as the efforts of their individual members and supporters, to survive even at the expense of a struggling civilization or a new civilization struggling through birth. In a similar vein, a great scientist once shot down the idea that scientists are so purely disinterested in their own ideas or institutions or other interests:

> A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it. [Max Planck, see `https://www.brainyquote.com/quotes/authors/m/max_planck.html`.]

So, even scientists hold on to their already formed worldview or parts of it, even when confronted by solid evidence they need to change their understanding of reality and maybe even some of their fundamental ways of thinking. Yet, science, for reasons well beyond the scope of this book, is self-corrective in certain ways that are substantial though also limited—scientists also draw upon worldviews outside of science and outside of its self-corrective processes. Young scientists can move on to new modes about thinking about some particular aspects or parts of this universe. There is no evidence whatsoever that scientists will 'rise above' the moral disorder of their age or even perceive, for example, the takeover of a once promising country by human beings with the moral character of gangsters. As I've noted before, scientists and engineers have helped to develop some of our horrible—outright evil—technology, nuclear bombs and lasers that permanently blind enemy soldiers and bombs that suck the lungs out of anyone nearby so they suffocate with their lungs hanging out of their mouths. It's true that they have also participated in the great rise in standard of living, that they have performed magnificently in exploring reality. This is no more than can be said for the blue-bloods of the northeastern United States. Read histories of the industrialization of the United States and then histories of the American part in the opium trade in China and you quickly find some families appearing in both; you will find comments (ironic?—hard to tell in the context of American thought) that these ambitious people,

founders of American industry and finance, weren't dedicated to crime but rather to getting a good rate of return on their invested capital and time. Scientists, as communities look for funding for work they enjoy and have been trained to do.

Why can these things happen? The moral failings of scientists, individuals and communities, are likely due to the same phenomenon that Planck referred to in the short quotation above. Scientists are indoctrinated in an instrumental ethics, a good one, that regulates how they do science, but it says nothing about the moral goodness or evil of their goals. Some of the physicists in the Manhattan Project realized after the first bomb was ready to be dropped that there were deep moral issues in creating and deploying and using such a weapon; too late—at least for some hundreds of thousands of Japanese as well as for the moral characters of many human beings who came to casually accept the use of this weapon as they had come to casually accept the dropping of explosive or inflammatory bombs upon civilian populations. More recently, there seems to have been some well-meaning medical researchers who were surprised to discover that some think there are moral issues involved in use of the flesh of human embryos, whether aborted or grown in the laboratory. Those scientists thought the only moral issues centered around their accepted duty to heal some terrible medical problems. I can understand and even admire their acceptance of that duty but also puzzle over the way in which those scientists were obvious to the moral beliefs of many in the West. Did they never so much as talk to one of their fellow-citizens who hold to the moral beliefs once shared by the grandparents of nearly all of us, scientists and non-scientists alike?

We all form worldviews and usually lay the foundations of moral character as children and adolescents followed by the development of a basic intellectual structure in our later adolescent years into our adulthood. Most human beings seem capable of building at this basic level but once and then their brains are set in their connections or their minds harden. Yet, all of us have holes in our worldview and nearly all of us have minds too rigid to truly conceive of our own holes when confronting others who don't have those particular holes. There are issues to be resolved in the best-intentioned of weapons development and medical research, but few are those who step out of their ruts to so much as survey the region they're traveling through. This rigidity, at least in the case of those who are properly leaders or science or religion or politics or other fields, can be at least partially eliminated by the type of thinking I've recommended in this book, thinking which acknowl-

edges the importance of abstract being and forces us—regarding important issues—to step up a level from our particular viewpoints and interests.

What happens when human communities begin to disintegrate, as the West is currently doing? What happens when the individuals and communities inside a civilization or a major religious community find their own children rejecting their beliefs and taking up with ways of thought and feeling and behavior forbidden not long ago, maybe not even so much explicitly forbidden as not even thinkable? What happens when those people find themselves and their children under pressure to live in ways which are in conflict with their beliefs?

Many will struggle to hold on to their seemingly outmoded beliefs, but I'm going to concentrate upon another group, those who struggle instead to make new sense of reality. If we look at recent history, during the various periods of turmoil, only a very small percentage of human beings engage in that second struggle but many follow some of those who propose new ideas, Lenin and Mussolini and Hitler and Gandhi and Mao and Martin Luther King, Jr and others. The phenomenon occurs in the purely technical side of mathematics and science as we can know from the above quotation from Max Planck but I've already claimed that science can be self-correcting over as little as a generation or so. Such a controversial idea as infinities greater than that of $\{1, 2, 3, \ldots\}$ can be absorbed—if not yet understood—by those just learning their multiplication tables at the time though perhaps not by those already tenured professors.

I'll make a few general comments on the difficulties encountered in politics and economics and philosophy and theology and many other fields without the self-corrective processes of science. They lack those institutional processes of self-correctiveness for various reasons including the fact that they involve—much more than mathematics or most sciences—entanglements of concrete and abstract being. The individuals in those fields outside the ones labeled 'science' in modern discourse also tend to have different attitudes toward knowledge than do those inside the privileged realm of modern science. They don't explore newly exposed lands so readily—scientists do explore, even the ones who sometimes reject what they discover. Non-scientists are not generally so appreciative of new information as are scientifically-minded human beings.

All of this new data, sometimes processed into forms plausibly labeled as 'information', is good stuff. It's good stuff that tells us much about God's Creation. It's good stuff that has led to serious contemplations about the

very nature of mathematics, contemplations which are one of the main inspirations of the worldview I'm trying to communicate in this book.

Let's consider the reformation of an existing worldview or changes so substantial as to bring about a new worldview. First of all, we have to realize that most human beings are not capable of doing the sort of work I'm doing, that of constructing a new worldview—just as I'm not capable of leading or administering a country or an army or a corporation. It would be a world of turmoil if too many were inclined to the sort of work I'm doing. It would also be a world lacking in important practical accomplishments, such as many of those which have led to our high standards of living and made it possible and necessary to think about these abstract issues. Because of this division of talents and responsibilities, most human beings will absorb a worldview and many of its parts and aspects—such as general attitudes—from their surroundings. This is still a matter of genius by the standards of simpler phases of human history, let alone the standards of non-human species. In the end, propaganda doesn't work because human beings might be willing to accept political and moral nonsense, especially in the context of a prosperous society such as the United States decreasingly is, but they do pay attention to their human and purely physical surroundings. This is what has so badly damaged membership in and strong belief in Christianity, whose institutions were the first to go out of synch with the modern world and its mountains of data only partially digested into information which is itself less fully digested into knowledge.

Now it has become obvious that our political communities are also out of synch with the best current understanding of those parts and aspects of this world which can be explored and analyzed by those labeled as 'scientists [1]'. In fact, nearly all human communities, even families and ethnic social clubs and sports associations, are out of synch with this confusion of data and information and knowledge which has destroyed our inherited understandings of even our universe, let alone our world (the universe as understood in light of moral order), let alone our Creation.

At least in the current situation of ongoing decay in the United States and the West as a whole, a worldview and its plausible variants decay into a multitude of ideologies of various sorts, most being implausible and ugly in various senses—moral and aesthetic and intellectual and political and so

[1] In premodern terms, science is simply any disciplined study and can include literary analysis and musical composition and old-fashioned homemaking.

on.

This process started centuries ago. Our current ideologies, New Left and Neoconservative and transhuman and trans-sexual and so on, are the ugly ideologies of the West, no more than the fragments and decay-products of the worldview of Western Christian civilization rigidified in recent centuries: the ideology of the nation-state as the center of human life or the overlapping ideology of the free-market as an absolute good are but two of those fragments or decay-products—two of the more plausible fragments at that, largely because they can be more easily integrated into a better and more complete worldview. What I refer to as the single worldview of Western Christian civilization (as it emerged from the wrongly vilified Medieval Period) was actually a spectrum of closely related variants; I'll ignore this complication as being unimportant to my main points.

I'll point to an interesting example of what I think to have been a promising political component to a better Western worldview: the American Old Right. The Old Right was largely non-ideological, in the sense I use it, because it was a mixed community of some who might nowadays be labeled as 'paleoconservatives' and of various sorts of morally conservative libertarians and even some who were inclined to old-fashioned village or local socialism. Despite the radical and incompetent (or perhaps dishonest) attacks upon the Old Right, it never congealed into an ideology as was happening in a very gradual way at first to New Deal liberalism and to certain branches of conservatism in the 1950s or so. As for the Trotskyites rebranding themselves as Neoconservatives, but keeping the taste for perpetual revolution itself rebranded as national security wars or wars to infect other countries with the American (corrupt, political-machine) political system, they are a strange and ugly story unto themselves.

So, how would I summarize my understanding of our current situation? I'll provide a mercifully simplified rendering of my complex understanding of recent intellectual and political history.

The Christians of the post-Medieval West, or Modern West if you prefer, failed to respond honestly and courageously and with faith in the Creator to the problems and opportunities raised by modern mountains of all that data and information and knowledge which came from exploration of the Earth's surface as well as the exploration of mathematical ideas and history and the behavior of light and so on. The worldview of the West began to decay into various strange and ugly ideologies, fragments of a badly deformed Western Christian worldview. These fragments ranged from:

- Western Catholic theology deformed as papal supremacy and Roman centralism of a degree certainly not recognized until after—not surprisingly—certain popes in way over their heads faced the loss of secular power. Not surprisingly, these exaggerated claims on behalf of the ecclesiastical descendant of the leader of the Apostles has resulted in various huge and unnecessary losses to Western Catholic Christianity.

- An implausible version of Christianity in the politicized form of reactionary aristocracy or monarchism.

- A still less plausible version of Christianity mixed with liberalism and concentrated by the Enlightenment into bloody and revolutionary democracy.

to the opposite extreme (by some understanding of 'opposite'):

- Economic determinism of socialist and capitalist types.

- Scientism allied with technocratism.

And, yes, I do consider Marxism and runaway capitalism as being found near the worship of science and technology on this spectrum of nasty and unsustainable ideologies.

All of the above systems, which I would describe as corrupted idealism or human ideas raised above empirical evidence, are closely related to good communal forms: Christianity or free-enterprise economies or science and technology. This would have to be the case because they are all fragments, decay-products of variants of Western Christian civilization (or communities inside of that greater community). It's not that they are directly decay-products of that great civilization. Rather is it the case that a set of closely related worldviews at the foundation of Western Civilization decayed into a set of closely related ideologies. As the decay continued, those closely related ideologies fragmented and continued to decay, producing new ideologies that were thoroughly psychotic—detached from reality.

12 A Short Wrapup

We need to do better if we are to deal with our problems. We may or may not be able to rescue Western Christian Civilization; we may or may not be able to build something new if that Civilization completes its ongoing and ugly collapse; if all else fails, we may or may not be able to contribute to a new Christian Civilization on the Pacific Rim or in Eastern Europe—the most likely centers for a new Christian Civilization in my opinion. We should try whether or not there be much chance of success.

What else do we have to do which is more important or more potential fun than exploring God's Creation with mind and heart and hands and then trying to understand and spiritually integrate and act upon what we discover? What could better prepare us to share God's life in Heaven?

Appreciating the glorious project in the previous paragraph, a project not to be completed in this mortal life, we can see that this is the best way to build a pilgrim Body of Christ—a civilization at least approximating the true Body of Christ, however incompletely and imperfectly. We can see that a Christian Civilization comes into being as it develops such a rich and complex understanding and feeling and doing of the acts-of-being which God manifested in Creation. We can see that we start this project only by coming to at least somewhat appreciate and nurture our communal human being. Only a group of communities approximating what is needed for a rich and complex civilization can build themselves into a singular human being, a communal human being of the best sort we can be in this mortal realm—one human being awaiting only its head, Jesus Christ, to be a perfect and complete human being.

Appendices

Bibliography

[1] A D Aleksandrov. *Mathematics: Its Content, Methods, and Meaning*, chapter Non-Euclidean Geometries. The MIT Press, 1969.

[2] St. Thomas Aquinas. *Commentary by St. Thomas Aquinas on the First Epistle to the Corinthians*. Ave Maria University Website, 1240. translated by Fabian Larcher, O.P. in unknown year and available as a printed book or as internet readable form at website of Ave Maria University.

[3] Hannah Arendt. *The Life of the Mind: The Groundbreaking Investigation of How We Think*. Harcourt, Inc., 1978. Published posthumously with minor editing by Mary McCarthy.

[4] Pierre Cartier. A mad day's work: From grothendieck to connes and kontsevich the evolution of concepts of space and symmetry. *Bulletin of the American Mathematical Society*, 38(4), 2001.

[5] Gregory Cochran and Henry Harpending. *The 10,000 Year Explosion: How Civilization Accelerated Human Evolution*. Basic Books, 2009.

[6] Pierre Duhem. *Medieval Cosmology*. The University of Chicago Press, 1985. Abridged version of multiple books first published in French 1913-1959.

[7] John Earman. *Bangs, Crunches, Whimpers, and Shrieks: Singularities and Acausalities in Relativistic Spacetimes*. Oxford University Press, 1995.

[8] Loyd Fueston. *Acts of Being*. Loyd Fueston, 2006. Main web-blog for discussing more technical issues of philosophy and science. Found at `http://loydfueston.com/`.

[9] Loyd Fueston. *To See a World in a Grain of Sand*. Loyd Fueston, 2006. Web-blog intended for discussing issues at a more popular level, including moral and religious implications. Found at `http://loydf.wordpress.com/`.

[10] Loyd Fueston. *Four Kinds of Knowledge*. Loyd Fueston, 2008. Downloadable at `http://loydfueston.com/downloads/know.pdf`.

[11] Loyd Fueston. *A Modern View of Creation: Making Peace with Empirical Reality*. Loyd Fueston, 2012. Downloadable at `http://loydfueston.com/downloads/acts-emp.pdf`.

[12] Loyd Fueston. *Acts of Being: Selected Weblog Writings from 2006 to 2012*. Loyd Fueston, 2013. Downloadable at `http://loydfueston.com/downloads/acts.pdf`.

[13] Loyd Fueston. *Catalog of Major Writings by Loyd Fueston*. Loyd Fueston, 2013. Downloadable at `http://loydfueston.com/downloads/about_books.pdf`.

[14] Loyd Fueston. *A More Exact Understanding of Human Being*. Loyd Fueston, 2013. Downloadable at `http://loydfueston.com/downloads/human-being.pdf`.

[15] Brad S Gregory. *The Unintended Reformation: How a Religious Revolution Secularized Society*. Harvard University Press, 2012.

[16] Ian Hacking. *Rewriting the Soul: Multiple Personality and the Sciences of Memory*. Princeton University Press, 1995.

[17] Allen Hatcher. *Notes on Introductory Point-Set Topology*. Allen Hatcher, Unknown. http://www.math.cornell.edu/ hatcher/Top/TopNotes.pdf.

[18] John G Hocking. *Topology*. Dover Publications, 1988.

[19] Donald W Kahn. *Introduction to Global Analysis*. Dover Publications, 2007.

[20] Marc Lachieze-Rey. *Cosmology: A First Course*. Cambridge University Press, 1995.

[21] A.D. Aleksandrov & A.N. Kolmogorov & M.A. Lavrent'ev. *Mathematics: Its Content, Methods, and Meaning.* The MIT Press, 1969. In 3 volumes.

[22] Charles Nash and Siddhartha Sen. *Topology and Geometry for Physicists.* Dover Publications, 2011.

[23] R. B. Onians. *The Origins of European Thought About the Body, the Mind, the Soul, the World, Time, and Fate.* Cambridge University Press, 1988.

[24] P.J.E. Peebles. *Principles of Physical Cosmology.* Princeton University Press, 1995.

[25] Carroll Quigley. *The Evolution of Civilizations: An Introduction to Historical Analysis.* Liberty Fund, 1979. A reprint.

[26] Bruno Snell. *The Discovery of the Mind in Greek Philosophy and Literature.* Dover Publications, 1982.

[27] Peter Turchin. *War & Peace & War: The Life Cycles of Imperial Nations.* Pi Press, 2006.

[28] C A van Peursen. *Body, Soul, Spirit: A Survey of the Body-Mind Problem.* Oxford University Press, 1966.

[29] various. Collaborative international dictionary of english v.0.48. Based upon Webster's Revised Unabridged Dictionary, 1913, C. & G. Merriam Co., Springfield, Mass and upon WordNet(R), a semantic network created by the Cognitive Science Department of Princeton University.

Colophon

This book was typeset using the LaTeX typesetting system created by Leslie Lamport and the memoir class written by Peter Wilson. The LaTeX typesetting system is a set of macro commands using the TeX typesetting system written by Donald Knuth. The body text is set 10/12pt on a 33pc measure with Computer Modern Roman designed by Donald Knuth. Other fonts include Sans, Smallcaps, Italic, Slanted and Typewriter, all from Knuth's Computer Modern family.